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RASCH MODEL ANALYSIS WITH THE BICAL COMPUTER PROGRAM

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The University of Chicago

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The Rasch Model is the mathematical formulation of any measurement situation, either physical or psychological, for which the relevant statistic can be expressed in terms of the number of successes (or number of targets hit, items correct or marks exceeded). The model specifies an efficient and reasonable way to make objective inferences from observations to variables. The concept and development of the model is discussed, emphasizing application to measurement of psychological variables. The BICAL program, a FORTRAN program to test

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the parameters, is described, with application to military police marksmanship data used as an illustration.

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1. THE NEED FOR OBJECTIVE MEASUREMENT

The Definition of Variables

Work in the behavioral sciences has been hampered by the notion that "measurement" has a different meaning for them than for the physical sciences. But it is fundamental in all scientific investigation to abstract from experience simple ideas which organize the complexity in useful ways. Useful ideas, often called "variables," are drawn from the scientist's careful observations of his experience, but they are necessarily over-simplifications intended to be meaningful for a particular purpose. Another scientist with other purposes may construct a different set of variables to summarize similar experiences. Ideas come to be generally regarded as "true" only when (and so long as) they are useful in predicting outcomes among an interesting class of possible events.

After supposing his variable, the scientist attempts to establish its definition by collecting, validating and calibrating observations that provide information about it. Once the observations with which to measure a variable have been specified and calibrated, the scientist has established an operational definition of that variable. He can then proceed in an orderly way to study the formulation of general principles about the processes involved and to predict the outcomes of other situations involving these processes.

The Measurement of Variables

Even carefully defined observations are of little interest in themselves. They are seldom chosen only for their own sake but rather for the information they contain about

the "variable", which is supposed to lie behind them. In order to extract this information we must attempt to specify explicitly the supposed relationship between observation and variable. It is the specification of this relationship that enables us to make inferences about the amount of the variable that each object possesses and so to make comparisons among objects based on the inferred variable.

The intent of this approach is to become free from the particular observations taken. If the observations are appropriate and the inferences correctly drawn, we want to need nothing else about them. We want to be able to make whatever comparisons we choose, among objects or among different occasions for the same object, regardless of which observations were made in each instance. Even though some observations are necessary to infer the amount of the variable present, once that is done, we want to be no longer bound to them.

These ideas can be illustrated with a simple example. A person entering a room might observe that he looks up to some people standing in the room and down to others. This might lead him to hypothesize the existence of a "height" variable. He might then decide to carry with him a stick with marks at various distances from the end and to observe for each person the number of marks exceeded. This would permit him to make judgements about the amount of height each person possesses that are more precise than "taller than me" or "shorter than me".

If the man developed a means for translating the number of marks passed into the height of the person, i.e. a model, it would be possible for him to compare any person's measure (i.e., the height inferred by the model from the number of marks passed) with any

other measure obtained from any other stick that has been connected to the same variable. The sticks need not be the same length nor have the same number of marks nor have the marks at the same intervals, so long as each has been properly connected to the variable "height".

In addition to freeing him from the necessity of always using the same (or identical) sticks, the model relating the observation to the variable must also provide him with the means for assessing the validity of the measurement. If a person is measured twice and the two measures are not the same, within statistical limits due to the precision of the instruments, he would conclude that the person has not been properly measured and without additional information would be at a loss to know which of the measures, if either, should be associated with the person.

Because the measurement permits the comparison of every new measure with all previous measures for the person, with a little experience, our observer could come to recognize characteristics of sticks and persons which lead to measures that persist from trial to trial. The measurement model is essential in this process because it provides a framework for recognising when an observation is surprising. If we know a person once passed say, 117 marks on some stick and we now observe that he passes 37 marks on another, we cannot tell if this is a surprising result unless both observations can be connected to the same fundamental variable. By knowing when to be alarmed, an observer can quickly learn, for example, that flexible round-ended sticks often give unpredicted results and that the height of people cannot be measured reliably when they are running or jumping.

Height is so familiar that we feel we can observe it directly. But, in fact, we cannot "observe" the height of an unfamiliar object when it is viewed in complete isolation. Like all other variables, our observation of height involves a series of comparisons of the unknown object with some available calibrated instrument.

The units of measurement for height are equally familiar and arbitrary. Their importance and usefulness is only because they have been defined and the definitions accepted by everyone who measures height. The statement that a person is six feet in height now specifies his height unambiguously with no further information required about how the measure was obtained. This was not the case when the standard of measure was the king's foot.

Psychological measurement is not different in principle from other kinds of measurement but at this point there is little consensus about what variables are important (i.e. useful, in general) and what units are convenient to measure them. The following example should help clarify the parallels between physical and psychological measurement.

An observer of military training might hypothesize the existence of a marksmanship variable, that soldiers vary in the amount of this variable that they possess and that they must possess a certain amount in order to be competent soldiers. (It should be noted that this last hypothesis goes beyond measurement. The consideration of how to determine the amount of the height variable that a person possesses did not involve decisions about how much he should have. Only after obtaining a satisfactory measure of the variable can we begin to investigate the relationship with other variables to establish what amounts of height or marksmanship are required for particular situations.)

One plan for studying marksmanship would be to follow each soldier through his career and observe when his level of marksmanship was adequate and when it was not. While at the end we would know a great deal about those particular soldiers, we would not be able to make comparisons among them, since it is unlikely that we would have comparable data for any two. It would be equally impossible to predict their success in any new situations with any degree of precision.

We would prefer to structure the situation so that observations relevant to marksmanship can be accumulated quickly, efficiently and economically. We mig decide that useful observations could be generated from the task of firing at a target on a practice range. While this obviously does not involve all factors that might be considered, it could be argued that it does contain an important element that is common to any situation for which marksmanship would be involved. Knowledge of the variable defined by the observation of firing at a target should enable us to make reasonable predictions about the outcome of more complex situations.

But the number of times the person succeeds in hitting the target is no more the measure of his marksmanship than is the number of marks passed on a stick a measure of his height. The number of hits will depend on the size, distance, etc of the target (i.e., its difficulty) as well as the person's skill. We require a model to remove the effect of target difficulty and to translate the observation into a measurement about the person. With this accomplished we no longer need worry about presenting identical targets to every person any more than we need to measure their height with identical rulers. All we need are calibrated targets.

Selection of the task and the measurement model are crucial. There is no reason to expect that every observation can be converted into the measure of the variable we

want or that every mathematical function that transforms discrete counts into continuous "variables" will be equally useful. In order to understand what is required of these, we need to develop more fully what it is reasonable to require of a "measurement."

The Requirements for Good Measurement

At the very least, a good measurement model should require that a valid test satisfy the following conditions:

1. A more able person always has a better chance of success on an item than does a less able person.
2. Any person has a better chance of success on an easy item than on a difficult one.

It follows from these conditions that the likelihood of a person succeeding on an item is the consequence of the person's position on the variable (his ability) and the item's position on the same variable (its difficulty) and that no other variables influence the outcome. This implies that the difficulty of an item is an inherent property of that item which adheres to it under all relevant circumstances without reference to any particular population of persons to whom the item might be administered.

A major consequence of these conditions is that it is possible to derive an estimator for each parameter that is independent of all other parameters. All information about a person's ability expressed in his responses to a set of items is contained in the simple unweighted count of the number of items which he answered correctly. Raw score is a sufficient statistic for ability. For item difficulty, the sufficient statistic is the number of persons in the sample who responded correctly to that item.

These common sense requirements enable us to formulate an explicit mathematical model and to use this model to assess the appropriateness of the observations for furnishing information about the variable we are seeking. These requirements are also deceptively demanding. Successful measurement depends on achieving sufficient control with respect to the observations taken, so that their variations differ only along a single variable. Even though persons differ in many ways, their measurement becomes possible only when one of these dimensions dominates the behavior prompted by the items administered. Even when items differ on a number of factors, they can be successfully used for measurement if the responses of persons can be dominated by only one of these factors. Thus measurement can succeed despite multidimensionality, when the multidimensionality is controlled so as not to be shared actively by both persons and items. Two examples should help make this clear.

Two types of items: Suppose we wish to measure "general mental ability" and to do this devise an instrument containing both reading comprehension and mathematical computation items. While this instrument is clearly two dimensional, measurement with it could succeed in situations where either

1. there is no person variable which affects the probability of success for the reading items differently than for the math items,

2. math ability and reading ability are so highly correlated in the population that they do not appear different.

In either case we should not care whether measurements were made entirely with reading items, entirely with math items, or any mixture in between, since all items measure the "same" variable. In the first case, there is only one variable (perhaps called "general

mental ability"). In the second, there are two but since they are so highly correlated a person high on one is high on the other. We can measure math ability with reading items and reading ability with math items, if we choose. It does not matter whether we call the resulting measure math, reading or general ability. However, if we try to assert that both types of items are necessary for a "fair" measurement and become involved in setting the correct proportion of each, we have admitted the multidimensionality of the situation and must instead measure the two variables separately with items appropriate to each. (If we are still interested in one number we could then argue about how the two measures could be combined into a single index.)

It is only possible to measure a person, who always has many different abilities, on one variable by carefully constructing an instrument which addresses just that one variable. We may sometimes get by with a multidimensional instrument, since the two alternatives above--one variable versus two highly correlated variables--are not distinguishable in data, but, when we use an instrument of items readily classifiable into two or more types, its (effective) unidimensionality must be corroborated with each new sample.

<u>One type of item with extraneous variables:</u> A contrasting case can be illustrated by considering the measurement of problem solving ability with an instrument composed of word problems. Proficiency on this instrument requires many abilities in addition to problem solving, not the least of which is the ability to read the language in which the problems are written. If the reading ability of every person is well above the readability of the problems, differences among items or persons in this respect will not affect performance on the instrument. However, if any person has difficulty reading

the problems, his measure of problem solving ability will be biased downward by this interfering factor. His probability of success will be influenced by the interaction between his reading ability and the readability of the problem. This can, of course, be eliminated by regulating readability to be well below the reading ability of the target population. Then, although the persons may still vary in their ability to read, variation in their scores on this instrument will be due to their variation in problem solving ability alone.

This case differs from the preceding one in that the items are of one type but each has a "difficulty" on two variables. As long as all persons are sufficiently able readers, the instrument can be used to measure problem solving ability. Theoretically, such an instrument could also be used to measure reading ability among very able problem solvers who were poor readers.

Random guessing on multiple-choice items is another instance of extraneous variation. Persons who are guessing succeed on difficult items more often than their abilities would predict. This makes them appear more able, when more difficult items are administered, since their frequency of success does not decrease as difficulty increases. A similar but opposite effect occurs when able persons become careless with easy items, making them appear less able than they are.

Such items "measure" two variables--the ability of interest and the tendency to guess or to become careless. The "guessingness" of the item may or may not be a simple function of the difficulty on the main variable but for the person two different variables are involved. The measure of either variable is threatened by the presence of the other.

These forms of multidimensionality have in common the attribute that different subsets of the items produce non-equivalent estimates of person ability and different subsamples of persons produce different estimates of item difficulties. This contradicts the requirements for good measurement specified in the Rasch model.

Unequal Item Discriminations: No discussion of disturbances in measurement is complete without mention of item "discrimination." Rasch's derivation of what is required in order to achieve objectivity (i.e., measures of person ability that are free of the sets of items administered, and calibrations of items that are free of the samples of persons used) leads to a model which rules out a parameter for item discrimination. If measurement objectivity is to be achieved, the situation must be arranged so that a parameter for discrimination is not necessary.

When the problem is approached from other perspectives, for example, when the observations are considered so valuable that the data are allowed to determine the form of the model, regardless of the effect on measurement, item discrimination is almost always included as a parameter. A model with a discrimination parameter (or any other additional parameter) will recover the observed data more completely than one without, but it is not at all clear when that is done what bearing the resulting "estimates" of discrimination can have on the generalizability and reproducibility of the situation. It remains to be settled whether discrimination "estimates" pertain to stable, meaningful parameters that are useful in characterizing future outcomes of similar situations or whether they are only temporarily useful as descriptive statistics for diagnosing trouble in one set of observations.

In theory, item discrimination is a measure of the amount of information an item contains about the quantity of the variable that a person possesses. In practice, it is better described as an index of the correlation over the sample of the item score with the operationally defined variable. These correlations can be "high" or "low" for the wrong reasons.

With the problem solving example, if the items vary in readability and their readability is near enough to the reading level of the persons so that some persons have difficulty reading some items, then these items will appear to vary in their power to discriminate along the problem solving scale, due to their connection to realability.

If the calibration sample is drawn from <u>one</u> population, items which no one is able to read will have no relationship to problem solving ability and items which everyone reads without difficulty will be the purest instances of the relevant behavior. Hence, the highest "discriminations" will be associated with items dominated by the variable of interest and the lowest will be for items most influenced by other factors.

However, if the calibration group consists of samples from <u>two</u> populations which have identical distributions of problem solving ability but differ in their reading ability, the group with the better readers will tend to score higher on the test. Then the items which are the most effective at separating the high and low scorers will be the items most influenced by readability. Therefore, in this instance, the items with the highest apparent discriminations will not be the ones with the strongest relationship to the variable of interest but rather the ones with the strongest relationship to the dimension along which the populations differ most, namely readability.

Models which include an item discrimination parameter will appear to "explain" data from either of these situations. Both, however, violate the undimensionality

assumptions employed by these other models, as well as by the Rasch model. Therefore, we are in the unfortunate position of having data which seem to fit the model although they do not comply with the requirements of unidimensionality. The Rasch model avoids this potential danger by uncovering unacceptable variation in discrimination and avoiding discrimination as an item parameter.

A situation for which it is sometimes argued that a discrimination parameter is legitimate is one in which the items vary in the amount of random fluctuation inherent in them. This is analogous to items differing in their factor loadings. But even in this case the requirements for good measurement given previously are not satisfied. Before we sacrifice this, we should consider what it means for items to differ in their inherent error and decide what it is reasonable to do about it.

It is difficult to construct examples of items that vary in information which cannot be explained by the presence of additional factors. One possible case might be an instrument containing both multiple-choice and completion items. They could both reflect the same variable but, since different behaviors are required, they might differ in their relationship to the variable. We might expect a completion item, which requires the person to recall and write in the correct answer, to discriminate more sharply than an item which only requires the person to recognise the answer. Recognition items give the person who does not recall or recognise the correct response the opportunity to eliminate responses he knows to be incorrect, thereby increasing his chances of choosing the correct one. If his success at this is related to his position on the latent variable, not to his "test-wiseness" or any other extraneous factor, intelligent guessing of this sort can provide information about the variable of interest.

But this problematic situation is easily avoidable by simply not mixing items requiring obviously different behaviors on the same elementary instrument. The influence of extraneous factors on the outcome is a problem for all response models. The Rasch model is less susceptible to this source of confusion, since it is not so readily adaptable to mixed influences. Previous research (Panchapakesan, 1969; Wright and Panchapakesan, 1969) indicated that the tests of fit for the Rasch model are sensitive enough to such disturbances to protect measurements from deterioration due to them.

The Rasch Logistic Response Model

George Rasch (1960) provided a rethinking of the measurement problem which overcomes most of the deficiencies of traditional analysis and avoids the theoretical complications of the other latent trait models. Rasch's stochastic response model describes the probability of a successful outcome of a person on an item as a function of only the person's ability and the item's difficulty. Using only the traditional requirement that a measurement be based on a set of homogeneous items monotonically related to the trait to be measured, Rasch derived his measurement model in the form of a simple logistic expression and demonstrated that in this form the item and person parameters are statistically separable. Andersen (1973a) elaborated and refined the mathematical basis for the model. Wright and Panchapakesan (1969) developed practical estimation procedures that made application of the model feasible.

Rasch's model, while based on the same requirement of the sufficiency of total score relied on by traditional methods, offers new and promising opportunities for advancing our understanding of measurement and departures from it. Since the parameters

of the model are separable, it is possible to derive estimators for each parameter independently of the others. The logistic transformation assigns an ability of minus infinity to a score of zero and plus infinity to a score of one hundred percent. This eliminates the bounds on the ability range and puts the standard errors of measurement into a reasonable relationship with the information provided by observed score. The tests of item fit which are the basis for item selection are sensitive to high discriminations as well as to low and so lead to the selection of those items which form a consistent definition of the trait and to the rejection of exceptional items. Finally, the explicitness of the mathematical expression of the model facilitates statistical statements about the significance of individual person-item interactions and so makes both a very general and a very detailed analysis of misfit possible.

The Rasch model provides an explicit framework for comparing observed with expected outcomes. The expected outcome of administering an item to a person is that predicted by the model assuming that the item is appropriate with respect to that person and that the person was adequately motivated to bring his full ability to bear on the item. The model permits us to assess the likelihood of the observed result, and hence, to make statements about the appropriateness of the particular item for the particular person.

Objective measurement eliminates many of the problems that have plagued test users. The Rasch model is both necessary and sufficient for objectivity in measurement. To best utilize the power of this model, we need to develop fully the concepts and mathematics related to it. Chapter II provides this development. The first section reviews the philosophy and concepts presented by Rasch and his students. Section

two derives the estimating equations for the Bernoulli (i.e. one trial per task) form and then generalizes to the Binomial form (several trials per task). Finally, goodness of fit tests are presented for assessing the adequacy of the calibration.

CHAPTER II

DEVELOPMENT OF THE RASCH MODEL

Rasch's development of his approach to measurement places central emphasis on the concept of "specific objectivity". (Rasch, 1960, 1961, 1967, 1968; Wright, 1968.) The problem of measurement is to make comparisons among two or more persons (more generally, "objects") or two or more items ("agents") using the information from the interaction of the objects with the agents. In psychometrics, we often begin by determining the characteristics of items based on an administration to a sample of people but our ultimate aim is to compare the performance of people on a set of items.

By "specific objectivity", Rasch means a comparison of any two persons, derived from a set of person-item interactions, which is independent of all item parameters and of all person parameters other than the two in question. Similarly, a statement about two items is independent of all person parameters and all other item parameters.

While such a property is highly desireable (Loevinger, 1947), it is not a natural consequence of person-item interactions but must be specifically built into the measurement process for every situation. The more natural circumstance is for every person to bring many abilities into every confrontation with an item. Unless the item is carefully constructed to tap only one of these abilities, the process will be governed by any number of person or item characteristics.

Rasch (1960, 1961) based his development of a measurement model on the following assumptions:

(a) the probability of a correct response to item i by person v is entirely governed by

$$P\{X_{vi} = 1|v,i\} = \frac{\theta_{vi}}{1 + \theta_{vi}} \qquad \theta_{vi} \geq 0$$

(b) in which the situational parameter θ_{vi} is the product of two factors

$$\theta_{vi} = \xi_v \epsilon_i$$

where ξ_v pertains to the person and ϵ_i to the item, and

(c) all answers, given the parameters, are stochastically independent.

It is clear from (a) that θ_{vi} represents the odds of success and from (b) that ξ_v is the ability of person v and ϵ_i is the easiness of item i.

The separability (also called "latent additivity") of the parameters shown in (b) makes possible objectivity in measurement. It follows from this that all information about a person's ability contained in his responses to a set of items is captured by the simple count of correct responses. This permits us to compare the abilities of two persons independently of the items administered.

Following Rasch (1960), the logarithm of the odds of success on item i by person v is:

(1) $$\log \frac{P_{vi}}{1-P_{vi}} = \log (\xi_v \epsilon_i) = \log \xi_v + \log \epsilon_i.$$

Therefore the abilities of person v and person u when observed on any item i can be compared, in logistic units, by subtraction:

(2) $$\Delta^i_{vu} = \log \xi_v + \log \epsilon_i - \log \xi_u - \log \epsilon_i = \log \xi_v - \log \xi_u$$

which does not involve the item parameter at all. Actually computing a number to estimate this difference requires us to make use of the sufficiency of total score. Since all the information about ability is contained in the number of correct responses, all persons who have the same score must be assigned the same estimated ability. Therefore, by grouping together the responses of all persons who scored r, we can obtain an estimate of P_{vi} for all v with scores or r:

(3) $$P_{ri} \approx \frac{X_{ri}}{N_r}$$ where P_{ri} is the probability of success on item i by persons with score r,

X_{ri} is the number of persons with score r who answered item i correctly, and

N_r is the number of persons with score r.

And so

(4) $$\Delta^{i}_{rs} \approx \log\left\{\frac{X_{ri}}{N_r - X_{ri}}\right\} - \log\left\{\frac{X_{si}}{N_s - X_{si}}\right\}$$

is the difference in ability between a person with score r and a person with score s, estimated with item i. Of course information is usually available from more than one item; statistical techniques which amalgamate the information from all into a single estimate are presented in the next section.

Since all parameters always appear in combination with at least one other parameter, there is an indeterminancy in the system that must be resolved before a particular estimate of a person's ability or an item's difficulty can be calculated. This can be done in many ways; a simple one is to select one item, say item 1, as the reference point and ...

log easiness to zero. This arbitrary choice does not affect the comparison of two persons in expression (2) but it makes it possible to compute a particular estimated ability for each person. From expression (1) the estimated ability, $\log \xi_v$ for score r is now equal to the log odds of success on item 1 for persons with score r.

$$(5) \qquad \log \xi_v \approx \log \left\{ \frac{X_{r1}}{N_r - X_{r1}} \right\} \quad \text{if } \epsilon_1 \equiv 1$$

Similarly values can be computed for all items other than item 1 by

$$(6) \qquad \Delta^r_{i1} = \log \xi_r + \log \epsilon_i - \log \xi_r - \log \epsilon_1 = \log \epsilon_i$$

where Δ^r_{i1} is the difference between the logit for item i in score group r and the logit for item 1 in score group r, or

$$\log \epsilon_i \approx \log \left\{ \frac{X_{ri}}{N_r - X_{ri}} \right\} - \log \left\{ \frac{X_{r1}}{N_r - X_{r1}} \right\} .$$

Once difficulties have been estimated in this fashion, we are able to compare two people, as in (2) above, who did not take the same item.

Andersen (1973a) provides the proof for the other side of specific objectivity. He shows that if raw score is taken as the sufficient statistic for ability, then the underlying model must be the Rasch model. It follows from this that the three assumptions given above are both necessary and sufficient for specific objectivity.

Wright (1968) introduced the terms "sample-free item calibration" and "test-free person measurement." This is not intended to imply that anything can be known about a person's ability without administering some items or that anything can be known about an item's difficulty without giving it to some persons. It does mean, as illustrated above,

that we can obtain estimates of ability that do not depend on the difficulties of the particular set of items we choose to administer. Any other set of appropriate items produce a statistically equivalent estimate for the person.

This avoids some troublesome problems for the test user. It solves the problem of form equating. Once a bank of items has been calibrated (i.e., the difficulty of each item estimated), any form made up of items from that bank has also been calibrated. Its ability estimates are on the common scale of measurement with no further manipulation. This was dramatically illustrated by Rentz and Bashaw (1975) who showed substantial savings in time and money through the use of Rasch techniques over traditional methods of form equating. The logical extension of this property suggests that each person can be administered a test tailored specifically for him and still measures can be obtained that are comparable for all (Wright, 1968, Wright and Douglas, 1975a).

Before presenting a discussion of some of the methods available for obtaining estimates of the model's parameter, we should mention that ability and difficulty will be expressed throughout in "logits" which are arbitrary units of measurement. A person's ability in logits is the natural log odds in favor of his succeeding on an item whose difficulty is at the origin on the scale. In other words, a person with ability 0.0 (i.e., ability equal to the difficulty of an item at the origin) has an even chance (odds 1 to 1) of succeeding on the item since log (1) = 0 or equivalently, from expression (1),

(7) $$1.0 = \frac{P_{vi}}{1-P_{vi}}$$

or

(8) $$P_{vi} = \frac{1}{2} .$$

Similarly, a person with ability 1.0 has about a three to one chance of success ($\log_e 3$ is approximately equal to 1.0). Logits are used because they are computationally convenient. In connection with this, we will use the notation:

$$\beta_v = \log \xi_v = \text{ logit ability for person } v$$
$$-\delta_i = \log \epsilon_i = \text{ logit difficulty for item } i.$$

Calibration of Item Parameters

Several methods of estimating item parameters, are treated in detail in Douglas (1975) and Wright and Douglas (1975b, 1976, 1977a, 1977b). They are reviewed below.

Conditional Maximum Likelihood Estimation: The mathematically ideal method is the conditional maximum likelihood approach which follows naturally from the separability of parameters. The estimates were derived in detail by Andersen (1973c). An approximation was developed by Wright (1966). Andersen's derivation begins with the Rasch model:

$$(9) \qquad P\{X_{vi}|\beta_v,\delta_i\} = \frac{\exp[X_{vi}(\beta_v-\delta_i)]}{1+\exp[\beta_v-\delta_i]} \qquad \begin{array}{l} X_{vi} = 0,1 \\ i = 1,L \\ v = 1,N \end{array}$$

If this model fully characterizes the interaction between person v and any item i the likelihood of a particular set of responses to L items, denoted by (X_{vi}), is

$$(10) \quad P\{(X_{vi})|\beta_v,(\delta_i)\} = \prod_i \left\{\frac{\exp[X_{vi}(\beta_v-\delta_i)]}{1+\exp(\beta_v-\delta_i)}\right\} = \frac{\exp\{r_v\beta_v\}\exp\{-\sum_i X_{vi}\delta_i\}}{\prod[1+\exp(\beta_v-\delta_i)]} .$$

This probability is seen to be composed of three parts: $\exp(r_v \beta_v)$ which connects the person's score and his ability; $\exp(-\sum_i X_{vi} \delta_i)$ which connects the data and the item parameters; and the denominator which involves no data.

The probability of observing a given raw score

(11) $$r_v = \sum_i^L X_{vi}$$

is the sum of the probabilities of all possible ways of obtaining the score r. That is,

(12) $$P\{\sum_i X_{vi} = r | \beta_v\ (\delta_i)\} = \sum \frac{\exp(\beta_v \sum_i X_{vi}) \quad (-\sum_i X_{vi} \delta_i)}{\prod_i [1 + \exp(\beta_v - \delta_i)]} \quad \text{for all } \sum_i X_{vi} = r$$

or

(13) $$P\{\sum X_{vi} = r | \beta_r, (\delta_i)\} = \frac{\exp(r_v \beta_v)\ \gamma_r}{\prod_i [1 + \exp(\beta_v - \delta_i)]}$$

where γ_r is an elementary symmetric function of the item difficulties which equals

$$\gamma_r = \sum_v [\exp(-\sum_i X_{vi} \delta_i)] \qquad \text{for all } \sum_i X_{vi} = r$$

and the summation is over all possible response vectors which sum to r.

The conditional probability of response vector (x_{vi}) given the raw score is found by dividing (10) by (13):

(14) $$P\{(X_{vi}) \mid r_v, (\delta_i)\} = \frac{\exp(-\sum_i X_{vi} \delta_i)}{\gamma_r}$$

which is an expression in the item parameters that is free of the ability distribution of the persons. This result depends on raw score being a sufficient statistic for ability.

The conditional likelihood of the entire data matrix $((X_{vi}))$, consisting of the L responses by each of the N persons, is:

$$(15) \qquad \Lambda = P\{((X_{vi})) \mid (r_v),(\delta_i)\} = \prod_{v} \left[\frac{\exp(-\sum_i X_{vi}\delta_i)}{\gamma_{r_v}} \right]$$

since the observations are stochastically independent given the parameters or their sufficient statistics. Letting $s_i = \sum_v^N X_{vi}$ and $\prod_r^{L-1} \gamma_r^{n_r} = \prod_v^N \gamma_{r_v}$ we have for the conditional likelihood:

$$(16) \qquad \Lambda = \frac{\exp(-\sum_i s_i \delta_i)}{\prod_v \gamma_r^{n_r}} .$$

Estimators for the (δ_i) are found by maximizing Λ in the usual way. Details of this and the iterative procedures necessary for obtaining estimates are given by Andersen (1972), Douglas (1975) and Wright and Douglas (1975b).

Unconditional maximum likelihood estimation: While formally correct, the conditional estimation techniques have serious practical problems. The computation of the elementary symmetric functions is quite expensive by the methods now used and incurs unacceptably large roundoff errors for tests of length greater than twenty items. Wright developed a less expensive technique using unconditional maximum likelihood which is reported in detail in Wright and Panchapakesan (1969) and Wright and Douglas 1975b). In their development, the unconditional likelihood of the data matrix is the double product of

P_{vi} over all persons and items. Thus,

$$(17) \qquad \Lambda = \prod^{L} \prod^{N} P\{X_{vi}|\beta_v, \delta_i\} = \frac{\exp[\sum_i \sum_v X_{vi}(\beta_v - \delta_i)]}{\prod_i \prod_v [1 + \exp(\beta_v - \delta_i)]}$$

or

$$(18) \qquad \Lambda = \frac{\exp(\sum_i \sum_v X_{vi}\beta_v) \exp(-\sum_i \sum_v X_{vi}\delta_i)}{\prod_i \prod_v [1 + \exp(\beta_v - \delta_i)]}$$

Again the responses are stochastically independent given the parameters. (The high correlations that are usually observed among a person's responses to a set of items are due entirely to their common relationship to the person's ability, β_v which the items are attempting to measure.)

The algebra for maximizing this likelihood is less complex if we work with the log likelihood:

$$(19) \qquad \lambda = \log \Lambda = \sum_v r_v \beta_v - \sum_i s_i \delta_i - \sum_i \sum_v \log [1 + \mathrm{EXP}(\beta_v - \delta_i)] + \varphi \sum_i \delta_i \;.$$

The final term is included to remove the indeterminancy in the equations that arises because only differences between parameters are estimable. The φ-term removes the problem here by imposing the restriction that $\Sigma \hat{\delta}_i = 0$. While almost any restriction on the δ_i would do this particular one is convenient for reasons to be discussed later.

The derivatives needed to obtain the maxima of (19) are:

$$(20) \qquad \frac{\partial \lambda}{\partial \beta_v} = r_v - \sum_i \frac{\exp(\beta_v - \delta_i)}{1 + \exp(\beta_v - \delta_i)} = r_v - \sum_i P_{vi}$$

$$(21) \qquad \frac{\partial \lambda}{\partial \delta_i} = -s_i + \sum_v \frac{\exp(\beta_v - \delta_i)}{1 + \exp(\beta_v - \delta_i)} + \varphi = -s_i + \sum_v P_{vi} + \varphi$$

$$(22) \qquad \frac{\partial^2 \lambda}{\partial \beta_v^2} = -\sum_i \frac{\exp(\beta_v - \delta_i)}{[1 + \exp(\beta_v - \delta_i)]^2} = -\sum_i P_{vi}(1 - P_{vi}) < 0$$

$$(23) \qquad \frac{\partial^2 \lambda}{\partial \delta_i^2} = -\sum_v \frac{\exp\ \beta_v - \delta_i)}{[1 + \exp(\beta_v - \delta_i)]^2} = -\sum_v P_{vi}(1 - P_{vi}) < 0$$

Wright and Douglas (1975b) demonstrated that the cross derivatives are small and can be ignored without harming the resulting estimates.

Since both second derivatives are always negative, there can only be one extreme point and it must represent the maximum likelihood. This point can be determined by setting equations (20) and (21) equal to zero and solving. We first need to evaluate φ. Summing equation (21) over all items,

$$(24) \qquad -\sum_i s_i + \sum_i \sum_v \hat{P}_{vi} + \sum_i \hat{\varphi} = 0$$

or

$$(25) \qquad -\sum_i \sum_v X_{vi} + \sum_i \sum_v \hat{P}_{vi} + L\hat{\varphi} = 0$$

and since from (20)

$$(26) \qquad \sum_i \sum_v X_{vi} = \sum_i \sum_v \hat{P}_{vi} \ .$$

We must have that $\hat{\varphi} = 0$. The estimating equations are simply:

$$s_i = \sum_r n_r \hat{P}_{ri} = \sum_r n_r \left[\frac{\exp(b_r - d_i)}{1 + \exp(b_r - d_i)} \right] \tag{27}$$

and

$$r = \sum_i \hat{P}_{ri} = \sum_i \left[\frac{\exp(b_r - d_i)}{1 + \exp(b_r - d_i)} \right]. \tag{28}$$

We are able to substitute an r subscript for the v-subscript in (27) because r is a sufficient statistic for ability so persons who attain the same score are indistinguishable as far as our knowledge of their ability is concerned. It is more efficient to perform the summations from 1 to L-1 rather than 1 to N.

Since (27) and (28) can not be solved explicitly for b_r and d_i, we must resort to an interative solution. The simplified Newton-Raphson approach given by Wright and Panchapakesan (1969) works quite well for this.

$$d_i^{t+1} = d_i^t - \frac{s_i - \sum_r n_r P_{ri}^t}{\sum_r n_r P_{ri}^t (1 - P_{ri}^t)} \tag{29}$$

and

$$b_r^{t+1} = b_r^t + \frac{r - \sum_i P_{ri}^t}{\sum_i P_{ri}^t (1 - P_{ri}^t)} \; . \tag{30}$$

The meaning of these expressions can be grasped intuitively by noting that the numerator of each correction term (i.e., the right hand terms) are equations (24) and (25). When this term is zero, the equation is solved and we no longer need modify the estimates. If it is not zero, we adjust the estimate by an amount proportional to this

difference. The scaling factor in the denominator is the derivative of the P_{ri} with respect to the parameter, the change in scale from score units to logit units.

Starting values needed to begin the process can be obtained by computing the d_i assuming the b_r are zero and analogously, the b_r assuming the d_i are zero. From (27) we have

$$s_i = \sum_r n_r \left(\frac{\exp(-d_i)}{1+\exp(-d_i)} \right) = N \left[\frac{\exp(-d_i)}{1+\exp(-d_i)} \right] \tag{31}$$

or

$$d_i^o = \log \left(\frac{N - s_i}{s_i} \right). \tag{32}$$

From (28) we obtain

$$b_r^o = \log \left(\frac{r}{N-r} \right) . \tag{33}$$

It is clear from any of the estimation equations that zero or perfect scores for either persons or items can not be used to estimate parameters. In (32) and (33), this would lead to either zero or infinity for which the log function is not defined. In (29) and (30), the process could not converge unless all P_{ri} were zero or one, which can not happen unless the abilities or difficulties are plus or minus infinity.

In light of this, the first step in the estimation process must be the elimination of zero and perfect scores. This process may require more than one cycle since the elimination of an item which every one answered correctly necessitates the elimination of all persons who only answered that one item correctly, and so forth.

A second problem is that the *unconditional maximum likelihood* estimates are biased (Andersen, 1973b). For the case of a two item test it can be shown that the difficulties are biased by a factor of two. Wright and Douglas (1975b), based on earlier work by Wright (1966), demonstrate that for tests of any length L for which $\Sigma d_i = 0$ the average bias is (L/L-1) and that correcting all difficulties by this factor results in estimates that are virtually indistinguishable from those given by the more expensive but unbiased conditional estimation procedure.

The corrected unconditional estimation algorithm employed by most Rasch analysis programs (e.g., Wright and Mead, 1975) is

i) Obtain item scores, (s_i), and counts of the number of persons at each score, (n_r).

ii) Edit these data vectors to remove perfect scores (i.e., $s_i = 0$ or N and r = 0 or L) cycling as many times as necessary.

iii) Define an initial set of (b_r) as

$$b_r^o = \log\left(\frac{r}{L-r}\right), \quad r = 1, L-1$$

iv) Define an initial set of (d_i) as

$$d_i^o = \log\left(\frac{N-s_i}{s_i}\right), \quad i = 1, L$$

Center the item set at zero by subtracting $d. = \Sigma d_i / L$ from each d_i.

v) Obtain a revised set of (d_i) by the one dimensional Newton-Raphson alogrithm until convergence is achieved.

vi) Using the tentative set of (d_i) as obtained from (v) above, obtain a revised set of (b_r) once again by Newton Raphson.

vii) Repeat steps (v) and (vi) as often as necessary to obtain stable values for the (d_i).

viii) Correct for bias by multiplying each d_i by $(L-1)/L$.

ix) Calculate the approximate (b_r) for these unbiased (d_i).

<u>Cohen's normal approximation:</u> As a final alternative to the problem of estimating item difficulty parameters, Wright and Douglas (1975b) present the details to a very inexpensive procedure that was suggested by Cohen in 1973. This procedure assumes that person abilities are given by an explicit function of total score, and that the function is completely determined except for a single multiplying parameter which can be obtained by maximum likelihood. This implies that the distribution of both person abilities and item difficulties are adequately characterized by the first two moments. If this is true, the resulting estimates are identical to those obtained by the more expensive procedures just discussed.

The procedure is as follows:

i) Define the initial values of difficulties and abilities and their variances in the sample:

$$d_i^o = \log\left(\frac{N-s_i}{s_i}\right) - d_\cdot^o \quad \text{where } d_\cdot^o = \Sigma d_i/L$$

$$D = \sum^{L} d_i^{o\,2}/[(L-1)(2.89)]$$

$$b_r^o = \log\left(\frac{r}{L-r}\right)$$

$$B = \sum_r n_r (b_r^o - b_\cdot^o)^2/[(N-1)(2.89)] \quad \text{where } b_\cdot^o = \sum_r n_r b_r^o/N$$

$$r = 1, L-1$$

ii) Compute the expansion coefficients:

$$X = [(1 + B)/(1 - BD)]^{1/2}$$

$$Y = [(1 + D)/(1 - BD)]^{1/2}$$

iii) Compute the final estimates of the parameters and their standard errors:

$$d_i = Xd_i^o \tag{34}$$

$$SE(d_i) = X[N/s_i(N - s_i)]^{1/2}$$

$$b_r = Yb_r^o$$

$$SE(b_r) = Y[L/r(L - r)]^{1/2} \; . \tag{35}$$

Although there is only modest experience with this form of the alogorithm evidence indicates that for moderately long instruments and more or less symmetrical, unimodal score distributions, it yields estimates well within a standard error of the values obtained from the more expensive methods.

Binomial Extension of the Simple Logistic Model

Not all data is scored dichotomously. However, the ideas and equations of the preceding sections can be extended to more complex cases. Consider a situation in which a subject v receives a score of 0, 1, ..., m_i on an item i. This might be a score on an attitude scale, an aptitude test, or target shooting. If this score is taken to be generated as the result of m_i independent Bernoulli trials, each with probability of success P_{vi}, then the binomial response model

$$P(X_{vi}|P_{vi}, m_i) = \binom{m_i}{X_{vi}} P_{vi}^{X_{vi}} (1 - P_{vi})^{m_i - X_{vi}} \tag{36}$$

describes it (Andrich, 1975). In a given situation we may not be certain that this model (or the specialization we propose) is appropriate, but we can test the fit of the model once the parameters have been estimated.

It is useful to write this model in odds notation by letting

$$P_{vi} = \lambda_{vi}/(1 + \lambda_{vi}) \tag{37}$$

where λ_{vi} is interpreted as the odds of success. Then

$$P\{X_{vi} | \lambda_{vi}, m_i\} = \binom{m_i}{X_{vi}} \lambda_{vi}^{X_{vi}} / (1 + \lambda_{vi})^{m_i} \quad . \tag{38}$$

By analogy to Rasch's simple logistic model[1] it seems likely that it will be useful to write

$$\lambda_{vi} = \xi_v \epsilon_i \; . \tag{39}$$

That is, each λ_{vi} will be taken to be the product of a person parameter ξ_v and an item parameter ϵ_i . With this assumption we have

$$P\{X_{vi} | \xi_v, \epsilon_i, m_i\} = \frac{\binom{m_i}{X_{vi}} (\xi_v \epsilon_i)^{X_{vi}}}{(1 + \xi_v \epsilon_i)^{m_i}} \quad . \tag{40}$$

Note that if $m_i = 1$, then X_{vi} is zero or one and expression (40) reduces to the Bernoulli form of the preceding section.

Conditional Estimation

Let us consider the possibility of estimating the parameters ξ_v and ϵ_i . The model

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[1]The notation is somewhat less complex if we define: $\xi_v = \exp(\beta_v)$ and $\epsilon_i = \exp(-\delta_i)$.

(40) implies as usual the inevitable assumption of conditional independence of responses over persons and over items, given the parameters of the model. Suppose, then, that a person responds to L items. By our assumption of conditional independence, the probability that his responses will be $X_{vi}, \ldots, X_{vL}$ (which we shall write as (x_{vi})), given the parameters, is

$$(41) \qquad P\{(X_{vi}) | \xi_v, (\epsilon_i), (m_i)\} = \frac{\prod_i \binom{m_i}{X_{vi}} \xi_v^{\sum_i X_{vi}} \prod_i \epsilon_i^{X_{vi}}}{\prod_i (1 + \xi_v \epsilon_i)^{m_i}}$$

where (ϵ_i) and (m_i) represent $\epsilon_1, \ldots, \epsilon_L$ and $m_1, \ldots, m_L$ respectively. If we now denote the total score for any person as

$$(42) \qquad r_v = X_{v+} = \sum_i^L X_{vi}$$

then

$$P\{r_v | \xi_v, (\epsilon_i), (m_i)\} = \sum_{r_v} P\{X_{vi} | \xi_v, (\epsilon_i), (m_i)\} = \frac{\prod_i \binom{m_i}{X_{vi}} \xi_v^{r_v} \prod_i \epsilon_i^{X_{vi}}}{\prod_i (1 + \xi_v \epsilon_i)^{m_i}}$$

which can be rewritten as

$$(43) \qquad P\{r_v | \xi_v, (\epsilon_i), (m_i)\} = \frac{\xi_v^{r_v}}{\prod_i (1 + \xi_v \epsilon_i)^{m_i}} \prod_i \left[\binom{m_i}{X_{vi}} \epsilon_i^{X_{vi}} \right]$$

where the sum is taken over all collections of responses $(x_{v1}, \ldots, x_{vL})$ such that $x_{v1} + \ldots + x_{vL} = r_v$.

The conditional probability of a particular set of responses (X_{vi}) can be found by dividing (41) by (43) and, observing that the probability is now independent of δ_v.

(44) $$P\{(X_{vi})|r_v,(\epsilon_i),(m_i)\} = \frac{\prod_i \binom{m_i}{X_{vi}} \epsilon_i^{X_{vi}}}{\Sigma\left[\prod \binom{m_i}{X_{vi}} \epsilon_i^{X_{vi}}\right]}$$ where the summation in the denomination is over all persons with score r.

Clearly r_v is a sufficient statistic for ξ_v and $s_i = X_{+i}$ is a sufficient statistic for ϵ_i so all the information about a person's ability or an item's difficulty is contained in the appropriate total score. Furthermore, given a group of persons it is now possible in principle to compute the conditional likelihood of their responses and to estimate the item difficulty parameters by conditional maximum likelihood estimation independently of the abilities. Similar' , abilities could be estimated independently of item difficulties.

Details of the conditional maximum likelihood estimation procedure for the simple logistic case (all $m_i = 1$) can be found in Wright and Douglas (1975b). Unfortunately, the conditional maximum likelihood estimation is quite sensitive to round-off errors; even an improved estimation procedure which Wright and Douglas devised failed for moderate numbers of items. There is no reason to believe that conditional estimation would be more practicable in the binomial case.

Unconditional Estimation

Even if the conditional estimation procedure could be made to work, its excessive cost would probably inhibit wide application. Recognizing the cost and instability of

conditional estimation, Wright and Panchapakesan (1969) proposed a method of joint parameter estimation for the simple logistic model. This estimation procedure has been extended to the binomial case.

Let $((X_{vi}))$ be the matrix of responses of persons 1, ..., N to items 1,..., L, that is,

(45) $$((X_{vi})) = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1L} \\ X_{21} & X_{22} & \cdots & X_{2L} \\ \vdots & \vdots & & \\ X_{N1} & X_{N2} & \cdots & X_{NL} \end{bmatrix}$$

By conditional independence we have the joint probability

(46) $$\Lambda = P\{((X_{vi}))\mid(\xi_v),(\epsilon_i)\} = \frac{\prod^N \prod^L \binom{m_i}{X_{vi}} (\xi_v \epsilon_i)^{X_{vi}}}{\prod^N \prod^L (1+\xi_v\,\epsilon_i)^{m_i}},$$

so

$$\lambda = \log \Lambda = \sum_i \sum_v \log \binom{m_i}{X_{vi}} + \sum_v r_v \log \xi_v + \sum_i s_i \log \epsilon_i$$

$$- \sum_i \sum_v m_i \log (1 + \xi_v\, \epsilon_i).$$

Writing $\beta_v = \log \xi_v$, $\delta_i = -\log \epsilon_i$ as in the simple logistic case gives

(47) $$L = \sum_i \sum_v \log \binom{m_i}{X_{vi}} + \sum_v r_v \beta_v - \sum_i s_i \delta_i - \sum_i \sum_v m_i \log(1 + \exp(\beta_v - \delta_i)).$$

Thus

(48) $$\frac{\partial\lambda}{\partial\beta_\mu} = r_\mu - \sum_i m_i P_{\mu i}, \quad \mu = 1, \ldots, N,$$

(49) $$\frac{\partial^2\lambda}{\partial\beta_\mu^2} = -\sum_i m_i P_{\mu i}(1 - P_{\mu i}), \quad \mu = 1, \ldots, N,$$

(50) $$\frac{\partial\lambda}{\partial\delta_i} = -s_i + \sum_v m_i P_{vi}, \quad i = 1, \ldots, L.$$

and

(51) $$\frac{\partial^2\lambda}{\partial\delta_i^2} = -\sum_v m_i P_{vi}(1 - P_{vi}), \quad i = 1, \ldots, L.$$

Recall that all subjects with score r will have the same estimated ability $b_r = \hat{\beta}_{v_r}$ so equations (48) lead to the estimation equations

(52) $$r - \sum_i m_i P_{ri} = 0, \qquad r = 1, \ldots, M - 1$$

where $d_i^o = log[(N-S_i/Si]$. Observe that (52) has no solutions for zero and perfect scores, so they must be eliminated from the data. Similarly, (50) gives the estimation equations

(53) $$s_i - \sum_r n_r m_i P_{ri} = 0, \qquad i = 1, \ldots, L$$

where n_r is the number of subjects with score r.

Our experience with the simple logistic model leads us to expect a dependency in these equations, and, indeed, summing (52) over r and (53) over i gives identical sums. We resolve this dependency by setting $\sum_i m_i d_i = 0$. Other constraints might be used,

but this one helps keep down rounding error during estimation; and other linear constraint can be implemented by transforming the parameter estimates obtained using this one.

It is now a simple matter to estimate the β_v and δ_i by the Newton-Raphson method. Details of the estimation process for the simple logistic case can be found in Wright and Douglas (1975b) or Wright and Mead (1975). Their procedure generalizes directly to the binomial case.

Andersen (1973a) has shown that these estimates are biased. However, Wright and Douglas (1975b) show by simulation that most of the bias can be cleared up by multiplying the d_i by $(L-1)/L$ when all $m_i = 1$. Further simulation indicates that $(M-1)/M$ is a suitable unbiasing constant for the binomial case.

Standard Errors

In principle, asymptotic estimates of the standard errors of the parameter estimates are given by

$$SE(\theta) = [-\operatorname{diag}\{(\partial^2\lambda/\partial\theta^2)^{-1}\}]^{1/2} \quad .$$

Here the matrix of second derivatives is nearly diagonal, so we take

$$(54) \qquad SE(d_i) = (-\partial^2\lambda/\partial\delta_i^2)^{-1/2}$$
$$= \left(\sum_r^{M-1} n_r m_i P_{ri}(1-P_{ri})\right)^{-1/2}$$

and

(55) $$SE(b_r) = (\Sigma\, m_i\, P_{ri}(1 - P_{ri}))^{-1/2} \; .$$

Tests of Fit

A primary benefit from having an explicit mathematical model for a process is the possibility of making rigorous tests of how well the observed data are predicted by the model. In the case of the Rasch model, the most detailed form of the data is an N x L matrix, denoted by $((X_{vi}))$ consisting of one row for each person and one column for each item. The entry X_{vi} is the score of person v on item i. It has a range of 0 to m_i. For the most familiar Bernoulli form of the model, all m_i are equal to one.

The expected value of X_{vi} is

(55) $$E(X_{vi}) = m_i\, P_{vi}$$

and its variance is

(56) $$V(X_{vi}) = m_i\, P_{vi}(1 - P_{vi}).$$

Therefore, the difference between the observed score for the person and the predicted score

(57) $$x_{vi} = X_{vi} - m_i\, P_{vi}$$

may be standardized by dividing by the estimated standard deviation.

(58) $$Z_{vi} = \frac{X_{vi} - m_i\, P_{vi}}{(m_i P_{vi}(1 - P_{vi}))^{1/2}} \; .$$

The sample-free property of the model suggests one strategy for organizing the residuals. Since the estimates of item difficulty should, under the model, be independent of the distribution of person ability, the difficulty estimator should be equally appropriate for all scores. In other words, we should obtain the same estimated difficulty when just the low scores are used as when the high scores are used. If we were to adjust the estimates to fit score r exactly the first adjustment for item i would be proportional to (compare to expression (29))

$$x_{ri} = \sum_{v \epsilon r} X_{vi} - n_r m_i P_{ri} \quad . \tag{59}$$

If we standardize by dividing by the standard deviation and square

$$V_{ri}^2 = \left[\frac{\left(\sum_{v \epsilon r} X_{vi} - n_r m_i P_{ri} \right)^2}{n_r m_i P_{ri} (1 - P_{ri})} \right] K . \tag{60}$$

We obtain a chi-square statistic with one degree of freedom. The multiplier K is a correction factor, usually near one, to inflate the statistic to the equivalent of one degree of freedom. (Haberman, 1973). If all the n_r are equal and $P_{ri}(1-P_{ri})$ is nearly constant for all r and i, then K can be shown to be:

$$K = \frac{L(M-1)}{(L-1)(M-2)} \quad . \tag{61}$$

The intuitive motivation for this can be grasped easily by noting that since i goes from 1 to L and r from 1 to M-1, there are L(M-1) statistics V_{ri}^2. But, having fit L-1 item paramet and M-1 person parameters, there are only (L-1)(M-2) degrees of freedom available.

Since it frequently happens that some scores are not observed in a particular sample, or are very rare, the summation may also be done over score groups containing more than one score:

(62) $$V_{ji}^2 = \left[\frac{\left(\sum_{v\epsilon j} X_{vi} - \sum_{r\epsilon j} n_r m_i P_{ri}\right)^2}{\sum_{r\epsilon j} n_r m_i P_{ri}(1-P_{ri})}\right] \times K$$

$$K = \frac{Lg}{(L-1)(g-1)}$$

$$i = 1, L$$
$$j = 1, g \quad .$$

Collecting over groups, $V_{gi}^2 = \sum_{j\epsilon g} V_{ji}^2$ gives a chi-square statistic with g degrees of freedom.

While V_{gi}^2 specifically asks the question would all score groups give the same estimate of difficulty for item i, it is possible to compute a more general statistic from expression (58). Squaring and summing over all persons gives a test statistic for the fit of item i:

(63) $$NV_i^2 = \sum_v Z_{vi}^2 \left[\frac{NL}{(M-1)(L-1)}\right]$$

which is approximately distributed as a chi-square with N degrees of freedom. V_i^2 will tend to be large when different groups give different estimates of abilities (as will V_{gi}^2) and when persons in the same score group obtain their score in different ways.

A frequently mentioned alternative to the Rasch model is the logistic model containing a second item parameter, *item discrimination*. While this model lacks the essential measurement properties of the Rasch model, it can help conceptualize misfitting data if an index of discrimination is computed. Such an index can be derived as follows. General expression for the probability of the success of one trial is

$$(64) \qquad P(X_{vi} = 1) = \frac{EXP(\lambda_{vi})}{1+EXP(\lambda_{vi})} .$$

One possible parameterization of λ is that employed by the Rasch model; i.e., $\lambda_{vi} = \beta_v - \delta_i$. A possibility for an alternative generator might include a discrimination parameter. Then the probability would be as

$$(65) \qquad P(X_{vi} = 1) = \frac{EXP[\alpha_i(\beta_v - \delta_i)]}{1+EXP[\alpha_i(\beta_v - \delta_i)]} .$$

If this is the actual generator, then (64) and (65) are equal and the logits (the exponents in this application) are also equal, statistically, hence,

$$(66) \qquad \lambda_{vi} = \alpha_i\,(\beta_v - \delta_i) + e_{vi}$$

where the residual error e_{vi} is included, because the linear model cannot account for all the variation in λ_{vi}. Since expression (64) provides a unique parameter for each person-item combination, λ_{vi} is the same as the observed logit which in most application would be estimated by:

$$(67) \qquad \lambda_{vi} \approx \log\left[\frac{X_{vi}}{m_i - X_{vi}}\right] .$$

However, in our case, this cannot be done when X_{vi} is either zero or m_i.

To escape this, let us rewrite (66) in terms of a residual from the Rasch model:

$$\Lambda_{vi} = \lambda_{vi} - (\beta_v - \delta_i) = (\alpha_i - 1)(\beta_v - \delta_i) + e_{vi}. \tag{68}$$

The logistic residual Λ_{vi} can be approximated from (57) by recalling that the rate of exchange between score units and logits is approximately equal to the derivative of P_{vi} with respect to $(\beta_v - \delta_i)$. This derivative is

$$\frac{\partial P_{vi}}{\partial(\beta_v - \delta_i)} = P_{vi}(1 - P_{vi}) \tag{69}$$

and therefore,

$$\Lambda_{vi} \approx Y_{vi} = \frac{X_{vi} - P_{vi}}{P_{vi}(1-P_{vi})} . \tag{70}$$

Rewriting (68) in terms of statistics which we can compute, we have

$$Y_{vi} = a_i(b_r - d_i) + e_{vi} \tag{71}$$

where $a_i = (\alpha_i - 1)$. Since with respect to item i, the difficulty d_i is constant, an index of the item's discriminating power can be computed by regressing Y_{vi} on ability. Therefore,

$$a_i = \frac{\sum_v (b_r - b.)Y_{vi}}{\sum_v (b_r - b.)^2} \tag{72}$$

where $b. = \sum_r n_r b_r / N$

and the associated sum of squares is

$$SS(a_j) = \frac{[\sum_v (b_{r_v} - b)\, Y_{vi}]^2}{\sum_v (b_{r_v} - b)^2} \tag{73}$$

All the test of fit statistics presented in this section have the appearance of chi square (or mean square) variates, but recent simulation studies (Mead, 1976) show that this distribution is not exactly correct. Hence, exact probability statements about lack of fit are not possible. The chi-square distribution is a useful background against which to judge these statistics, however.

CHAPTER III

DESCRIPTION OF THE BICAL PROGRAM: ANALYSIS OF MILITARY POLICE PISTOL DATA

BICAL is a FORTRAN program designed to estimate and test the parameters in the Rasch model when written as:

$$(74) \qquad P\{X_{vi} | \beta_v, \delta_i, m_i\} = \binom{m_i}{X_{vi}} \frac{EXP[X_{vi}(\beta_v - \delta_i)]}{[1+EXP(\beta_v - \delta_i)]^{m_i}}, \quad X_{vi} = 1, m_i .$$

The observation X_{vi} represents the numbers of successes by person v in m_i trials at task i. The capacities of the program are listed in Table 1. The number of subjects permitted is restricted only by the availability of auxiliary storage. A description of the required control cards is given in Appendix C. The military police data will be used to illustrate the program's application.

The pistol data was collected to assess the competence of MP candidates. It involved eight target presentations which differed in the distance from the marksman and the position from which he was required to fire. For the first two targets, ten shots were required; for the remaining six, only five shots. The description of the task and number of shots at each are summarized in Table 2. Table 3 shows the control cards used for this analysis.

TABLE 1

BICAL PROGRAM CAPABILITIES

Description	Symbol	Maximum Value
Number of Items	L	150
Trials on one task	m_i	35
Total number of trials	$M = \sum_i m_i$	1000

TABLE 2

DESCRIPTION OF THE MILITARY POLICE PISTOL DATA

Task Number	Meters to Target	Position of Marksman	Task Name	Number of Shots
1	35	Prone	35P	10
2	25	Kneel	25N	10
3	25	Strong Left	25SL	5
4	25	Strong Right	25SR	5
5	15	Kneel	15N	5
6	15	Strong Left	15SL	5
7	15	Strong Right	15SR	5
8	7	Crouch	7C	5

TABLE 3

SAMPLE CONTROL CARDS FOR RUNNING BICAL

Card Number	Card Name	Card Format	Sample from MP data
1	Title Card	(20A4)	Military Police Data-Hits Per Target
2	Input Description	(14I5)	8 25 5 45 12 2 1
3	Item Names	(20A4)	35P 25N 25SL25SR15N 15SL15SR 7C
4	Column Select	(80A1)	AA555555
5	Key	(80A1)	77333333
6	Options Labels	(5A1)	12345
7	(Data Cards)		
7a	End of Data	(A1)	*
10	End of Job	(A4)	****

Card 1, is a title card which supplies identifying information to be printed at the top of each page of output.

Card 2, the data description card, describes how the data is to be presented and handled. The data for this run are described as follows.

First, there are eight tasks, i.e., each person has eight scores to be read, as described in Table 2. Second, the groups used in tests of fit must average at least 25 persons each. This determines the number of groups that will be used. The program format is limited to six groups, but if the total number of persons divided by six is less than twenty-five (as in this case) fewer groups will be used. This value will also halt the estimation of parameters if, after editing, there are fewer than twenty-five persons remaining in the sample. If no value is provided the default value is thirty.

The third and fourth values define the range of scores to be included in the calibration sample. Only persons scoring at least five but not more than forty-five will be included. This is done because extremely high or extremely low scorers frequently behave abnormally. The scores to be excluded need to be thought through for each application, for their choice depends on the way extreme scores might occur. In achievement testing, it is usually desirable to set the lower limit somewhat above the chance level.

Fifth, the value of "12" indicates that only the first twelve columns of each record need be read. Since in this case the data is punched in columns 5 through 12, there is no need to read beyond 12.

Sixth, the "2" selects the second available calibration technique. This is the corrected unconditional method, and is chosen for this problem because of the small sample size and the asymmetrical distribution of scores.

Seventh, the "1" specifies that the data is already scored and the value to be read for each item is the person's number of hits on that target.

All remaining parameters will assume default values. This means that data input is from cards, no output will be produced except on the printer and all standard printed output will be generated.

Card 3, the item name card provides a four character name for each item read. There are eight such fields here and they are coded in the same order as the items occur on the data cards.

Card 4, the column select card serves two functions. First, any character other than blank or zero indicates that the column contains an item included in the item count (8) on the data description card and named on the item name card. An ampersand (&) causes an item to be excluded from the analysis although read and named. This facilitates dropping misfitting items with a minimum of changes to the control cards.

The column select also defines the maximum possible score (m_i) for each item. Since the fields are only one column in width, the alphabetical characters (A-Z) are used to designate the values (10-35).[1] A value larger than 35 cannot be accepted by the program as it now stands.

The interpretation of the card given in Table 3 is that no data is wanted from columns 1 to 4 of the input record. Columns five and six contain tasks which have maximum scores of 10. Columns seven through twelve contain tasks, which have maximum scores of five. This accounts for all eight items.

[1] Data cards must be coded in the same way.

Card 5, the key card in Table 3 is not referenced in this run since the data is already scored (col. 35 of the data description card) but a key card must be included in the deck.

Card 6, the options label card defines up to five possible data values. The same five values apply to all items. The frequency of occurence of each of the specified values is accumulated for each item. For this example, a table showing the number of times each target was hit 1, 2, 3, 4 or 5 times will be prepared and printed.

Interpretation of BICAL Output for MP Pistol Data

The analysis offered here is intended to illustrate interpretation of the BICAL output; it is not a definitive analysis of this particular data set. Page 1 of the output shown in Appendix B lists the control cards just discussed. This enables the user to check quickly that the analysis performed is the one intended. In addition, the first input record and the total number of records are shown to verify that the records were read correctly.

Page 2 is the alternative response frequency table that was specified by the Options Label Card 6. The "UNKN" column is the count of the frequency of any character other than the five shown. Since targets 1 and 2 could have scores from zero to ten, these tasks show a large number of unknowns. For the others, the only unknowns are the zero scores.

Page 3 reviews the editing process. For this example there were no persons with zero or fifty hits. If there had been, these persons would have been excluded from all subsequent analyses. There were eight columns selected by the column select card and eight item names were provided.

There were no persons below five and ten above forty-five leaving a total of 126 to be used in the calibration. This table would not include the zero or perfect scores noted earlier.

No items were rejected because of perfect or zero item scores. Therefore, the analysis will be done on eight tasks with 126 persons. Had items and persons been eliminated the minimum and maximum accepted scores would have been suitably adjusted.

Pages 4 and 5 are histograms for person scores and item scores. For persons, the number at each score (i.e., number of hits) is shown. This is scaled to fill the grid with the scale factor shown at the bottom. For items, the figure shows the proportion of success for each item. For instance, there were 764 successes in 1260 trials on item one. The general impression given by these graphs is that the tasks were "relatively easy" for the persons resulting in high item scores, which increase as target distance decreases, and that there is a negatively skewed distribution of person scores.

Page 6 contains the difficulty estimates and the related standard errors of calibration for each item. These are the values needed for any future application of the items. The mean difficulty (weighted by the number of trials for each item) is always zero. As expected from the histogram, the difficulties decrease as target distance decreases. The standard errors are smallest for the most difficult tasks because of the high ability of the sample. These are the tasks with difficulties most like the abilities of the persons tested and hence for which the most information was obtained.

The table also provides some statistics on the estimation process. At the top the difficulty and ability, "scale factors" indicate the amounts by which the initial log odds estimates were inflated by the normal approximation method, "PROX".

The body of the table, in addition to the difficulty estimates and their standard errors, displays the magnitude of the adjustment in the last cycle (an indication of the rate of convergence), the difficulty estimate that was returned by "PROX" and the estimate after one cycle in "UCON". These are displayed to provide experience with how PROX compares to UCON and when the less expensive estimates are good enough. In this instance, there is little difference in the estimates even though the score distribution is skewed.

Page 7 gives the conversion of raw scores to estimated abilities and the standard errors of measurement associated with each score. The *test characteristic curve* is a picture of the range of ability covered by these eight tasks.

Pages 8 and 9 display a variety of item fit statistics. Unlike estimates of item difficulty, the tests of fit are very much sample dependent. That an item fits for one sample does not guarantee it will fit for another. Useful interpretation of these statistics requires both familiarity with them and a thorough understanding of the tasks and sample that generated them.

The basic statistic is the overall Fit Mean Square which appears on both pages (under the heading "total" on page 8). This is simply the mean squared standard residual z^2_{vi} averaged over persons. It will be large for an item if there are too many high ability persons who failed on the item and/or too many low ability persons who succeeded. What is "too large" depends on the requirements of the particular situa-

tion. The expected values and standard errors of these mean squares are 1 and $(2/f)^{1/2}$ where f is the number of groups. More than three standard errors greater than one seems to be a reasonable rule of thumb for "too large".

Two targets which have little else in common fall into this category. Target 1 is unique in several respects; it is the first in the sequence, it is at the greatest distance and is the only one that involves the prone position. All of these could be contributing to the misfit. Choosing among them would require a clinical investigation of the situation. This mean square says only that performance on this task has the weakest relation to performance on the other seven tasks. The non-significant between group mean square for this item (of 1.90) indicates that statistically equivalent estimates of difficulty would result from using either the low scorers or the high scorers for calibration.

Target 6 is not interesting in its position in the sequence and there were other targets at the same range and same position. This mean square is an index of the disagreement between the variable as defined by the item and the variable as defined by all items. The fit for this target which involved firing from the left could change if the mixture of shots from the right and left were changed. This would imply that an extraneous factor, handedness, has an influence on the outcome.

If we consider the possible effect of handedness on the difficulty of these tasks, the shots from the right side would tend to be easier for right-handed marksmen. Since eighty to ninety per cent of the sample would be right handed, a shot from the right would appear easier than the equivalent shot from the left. However, the effect is reversed for a left-handed marksman and this person would do poorly on the "easy" right handed shots and well on the "difficult" left handed shots. While this would

produce misfit over all person-task combinations, "surprising" results would tend to accumulate on the shots from the left because of the predominance of right-handed marksmen in the data. It might be eliminated by defining the shots as favored and not favored rather than left and right.

The between group mean square tests the agreement between the observed item characteristic curve and the best fitting Rasch characteristic curve as estimated by the groups selected. Five points on the observed curve are shown for each item on the left of page 8. The points shown were chosen by the program to represent groups of increasing ability and approximately equal size such that the average group size is at least 25.

The worst discrepancies between the curves are for targets 4 and 7, both of which involve firing from the right side. In particular, for target 4 score group two was seventy per cent successful while group three was only 62 per cent successful. The model predicted 64 and 73 per cent respectively. The discrepancy in proportion metric is given in the center panel of page 8. Complete understanding of the reasons for this requires greater knowledge of the effect of handedness on marksmanship but one hypothesis is that ability group three contained a preponderance of left-handed persons who do poorer than expected on shots from the right.

The remaining column on page 8 is the within group mean square. It is the misfit remaining after removing the effect of difference in the shape of the characteristic curves. It will be large and the between group effect small if the correct proportion of the group succeeded but the wrong people in the group were the ones who succeeded. It provides no information not contained in the between and the total but is a reorganization that is sometimes convenient.

The discrimination index is also closely related to the between group mean square. It is in fact the linear trend across score groups. Values larger than one indicate that the observed characteristic curve for an item is steeper than the average best fitting logistic curve for all items; values less than one indicate the curve is flatter. In this example there is no reason to suspect that the targets do not all have equal discriminations. In data simulated with exactly equal discriminations the standard deviation of the observed discriminations are frequently as large as 0.20, hence, the value observed here (0.11, from page 9) is quite acceptable.

Page 10 contains a plot of the discrepancies, standardized and squared, between the observed and fitted characteristic curves (center panel, page 8) against the probability of success for that group on that item. In this case, this plot does little to increase our understanding. It is useful with achievement tests where random guessing is a problem. In those situations large values of the z-squares are found near the chance level.

Pages 11, 12 and 13 are two-way plots of the three statistics given for each item on page 9; difficulty, discrimination and total fit mean square. There is no new information in them, but examining the plots is a convenient way to be certain not to miss any interesting results.

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APPENDIX A

BICAL OUTPUT PRODUCED BY THE MP DATA

```
MILITARY POLICE DATA--HITS PER TARGET                                                                                PAGE  1

 CONTROL PARAMETERS
NITEM NGROF MINSC MAXSC  LREC  NCAE SCORE    1    2    3    4    5    6    7    8    9   10   11
   8    25     5    45    12     2     1     0    0    0    0    0    0    0    0    0    1    3
COLUMNS SELECTED
         1         2         3         4         5         6         7         8
1********0*********0*********0*********0*********0*********0*********0*********0
    AA555555
KEY
    AA555555
FIRST SUBJECT
  1 75555555


NUMBER OF ITEMS    8
NUMBER OF SUBJT  136
```

```
MILITARY POLICE DATA--HITS PER TARGET                                                                                PAGE  2
```

ALTERNATIVE RESPONSE FREQUENCIES

SEQ NUM	ITEM NAME		1	2	3	4	5	UNKN		KEY
1	35P	I	4	6	6	17	13	90	I	
2	25N	I	7	3	8	10	18	90	I	
3	25SL	I	12	33	31	34	19	7	I	
4	25SR	I	7	10	28	47	39	6	I	
5	15N	I	2	12	17	37	68	0	I	
6	15SL	I	4	5	16	45	65	1	I	
7	15SR	I	2	1	4	29	100	0	I	
8	7C	I	0	0	7	24	105	0	I	

MILITARY POLICE DATA--HITS PER TARGET

```
NUMBER OF ZERO SCORES          0
NUMBER OF PERFECT SCORES       0

NUMBER OF ITEMS SELECTED   8
NUMBER OF ITEMS NAMED      8

--------------------------------
SUBJECTS BELOW      5          0
SUBJECTS ABOVE     45         10
SUBJECTS IN CALIB.           126
--------------------------------
TOTAL SUBJECTS               136

REJECTED ITEMS

 ITEM     ITEM   ANSWERED
NUMBER    NAME   CORRECTLY
----------------------------
          NONE

  SUBJECTS DELETED =    0
SUBJECTS REMAINING =  126

     ITEMS DELETED =    0
    POSSIBLE SCORE =   50

     MINIMUM SCORE =    5
     MAXIMUM SCORE =   45
```

MILITARY POLICE DATA--HITS PER TARGET   PAGE 4

SCORE   DISTRIBUTION OF ABILITY

SCORE	COUNT	PROPORTION	0 . . . 2 . . . 4 . . . 6 . . . 8 . . . 10
1	0	0.0	
2	0	0.0	
3	0	0.0	
4	0	0.0	
5	0	0.0	
6	0	0.0	
7	0	0.0	
8	0	0.0	
9	0	0.0	
10	0	0.0	
11	0	0.0	
12	0	0.0	
13	0	0.0	
14	0	0.0	
15	1	0.01	
16	1	0.01	
17	0	0.0	
18	0	0.0	
19	0	0.0	
20	2	0.02	XX
21	1	0.01	
22	0	0.0	
23	1	0.01	
24	2	0.02	XX
25	0	0.0	
26	3	0.02	XXX
27	1	0.01	
28	6	0.05	XXXXXX
29	5	0.04	XXXXX
30	3	0.02	XXX
31	4	0.03	XXXX
32	4	0.03	XXXX
33	5	0.04	XXXXX
34	0	0.0	
35	10	0.08	XXXXXXXXXX
36	8	0.06	XXXXXXXX
37	8	0.06	XXXXXXXX
38	9	0.07	XXXXXXXXX
39	3	0.02	XXX
40	13	0.10	XXXXXXXXXXXXX
41	9	0.07	XXXXXXXXX
42	9	0.07	XXXXXXXXX
43	8	0.06	XXXXXXXX
44	7	0.06	XXXXXXX
45	3	0.02	XXX
46	6	0.05	XXXXXX
47	2	0.02	XX
48	1	0.01	
49	1	0.01	
50	0	0.0	

EACH X = 0.75 PERCENT

MILITARY POLICE DATA--HITS PER TARGET PAGE 5

ITEM DISTRIBUTION OF EASINESS

ITEM	COUNT	PROPORTION	0 . . . 2 . . . 4 . . . 6 . . . 8 . . . 10
1	764	0.61	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
2	762	0.60	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
3	356	0.57	XXXXXXXXXXXXXXXXXXXXXXXXXXXX
4	444	0.70	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
5	516	0.82	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
6	518	0.82	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
7	552	0.92	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
8	553	0.94	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

EACH X = 2.00 PERCENT

MILITARY POLICE DATA--HITS PER TARGET PAGE 6

PROCEDURE USED UCON

DIFFICULTY SCALE FACTOR 1.115 ABILITY SCALE FACTOR 1.16

NUMBER OF ITERATIONS = 2

SEQUENCE NUMBER	ITEM NAME	ITEM DIFFICULTY	STANDARD ERROR	LAST DIFF CHANGE	PROX DIFF	FIRST CYCLE
1	35P	0.738	0.061	-0.001	0.765	0.739
2	25N	0.745	0.061	-0.001	0.772	0.747
3	25SL	0.526	0.065	-0.002	0.959	0.528
4	25SF	0.256	0.092	0.001	0.279	0.255
5	15N	-0.440	0.138	0.002	-0.441	-0.441
6	15SL	-0.463	0.109	0.002	-0.465	-0.465
7	15SF	-1.480	0.154	0.002	-1.543	-1.481
8	7C	-1.767	0.173	0.001	-1.656	-1.769

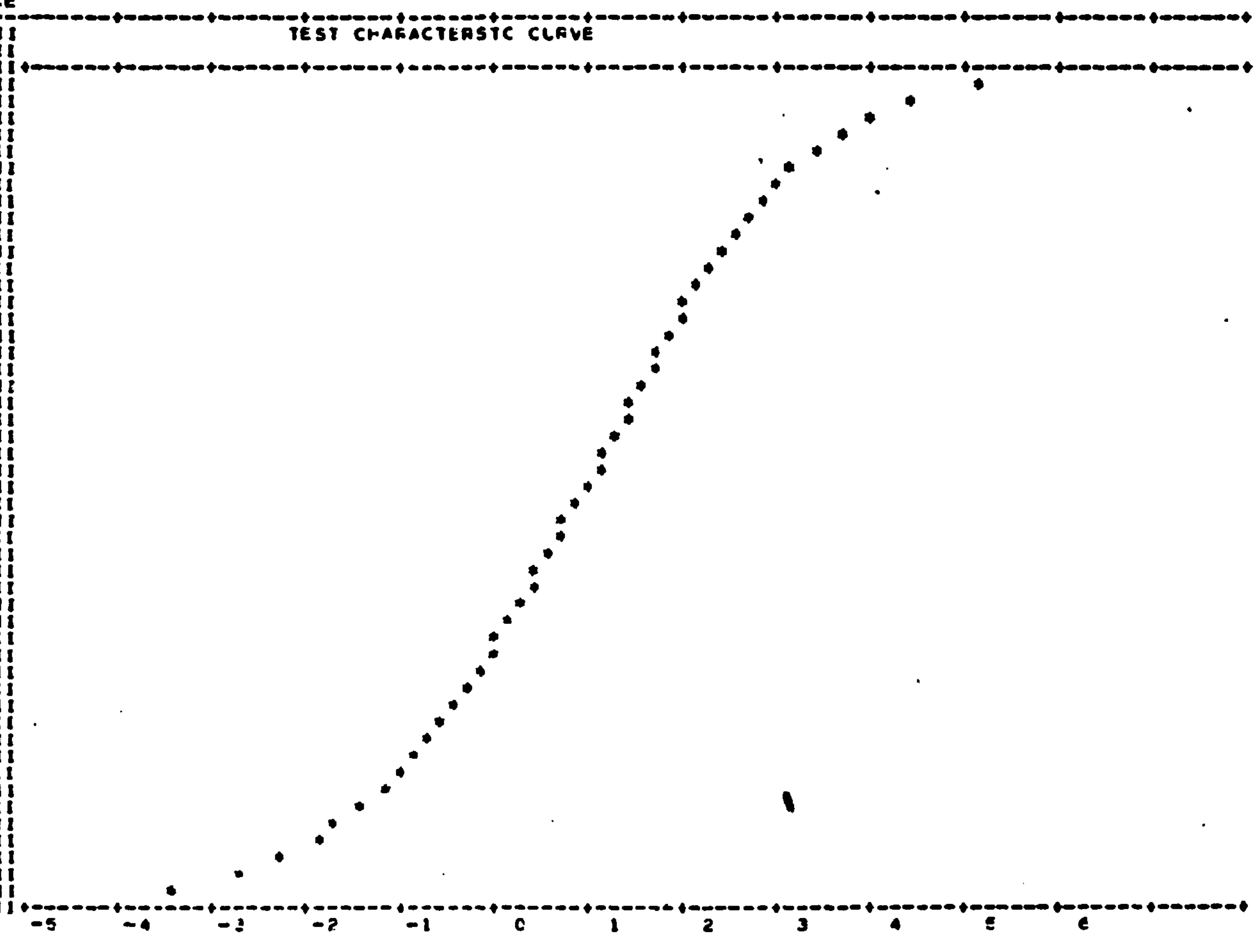
ROOT MEAN SQUARE = 0.002

MEAN ABILITY = 1.35
SD OF ABILITY = 0.90

COMPLETE SCORE EQUIVALENCE TABLE

RAW SCORE	COUNT	LOG ABILITY	STANDARD ERRORS
49	1	4.21	0.99
48	1	3.49	0.72
47	2	3.06	0.60
46	6	2.74	0.53
45	3	2.49	0.48
44	7	2.27	0.45
43	8	2.09	0.42
42	9	1.92	0.40
41	9	1.77	0.38
40	13	1.63	0.37
39	3	1.50	0.36
38	9	1.37	0.35
37	8	1.26	0.34
36	8	1.14	0.33
35	10	1.03	0.33
34	0	0.93	0.32
33	8	0.83	0.32
32	4	0.72	0.32
31	4	0.63	0.31
30	3	0.53	0.31
29	8	0.43	0.31
28	6	0.34	0.31
27	1	0.24	0.31
26	3	0.14	0.31
25	0	0.05	0.31
24	2	-0.05	0.31
23	1	-0.15	0.31
22	0	-0.24	0.31
21	1	-0.34	0.32
20	2	-0.44	0.32
19	0	-0.55	0.32
18	0	-0.65	0.33
17	0	-0.76	0.33
16	1	-0.87	0.34
15	1	-0.99	0.34
14	0	-1.11	0.35
13	0	-1.23	0.36
12	0	-1.36	0.36
11	0	-1.50	0.37
10	0	-1.64	0.39
9	0	-1.79	0.40
8	0	-1.96	0.42
7	0	-2.14	0.44
6	0	-2.34	0.46
5	0	-2.57	0.50
4	0	-2.84	0.54
3	0	-3.18	0.62
2	0	-3.63	0.74
1	0	-4.37	1.02

TEST CHARACTERSTC CURVE



MEAN ABILITY = 1.35
SD OF ABILITY = 0.66

MILITARY POLICE DATA--HITS PER TARGET

ITEM CHARACTERISTIC CURVE / DEPARTURE FROM EXPECTED ICC / FIT MEAN SQUARE

SEQ NUM	ITEM NAME	ICC 1ST GROUP	ICC 2ND GROUP	ICC 3RD GROUP	ICC 4TH GROUP	ICC 5TH GROUP	ICC 6TH GROUP	DEP 1ST GROUP	DEP 2ND GROUP	DEP 3RD GROUP	DEP 4TH GROUP	DEP 5TH GROUP	DEP 6TH GROUP	WITHN GROUP	BETWN GROUP	TOTAL	DISC INDX	POINT BISER
1	35P	0.32	0.51	0.65	0.71	0.79	0.0	-0.04	-0.02	0.06	-0.01	-0.01	0.0	1.52	1.90	1.53	1.05	0.65
2	25N	0.34	0.55	0.59	0.74	0.79	0.0	-0.01	0.02	-0.03	0.03	-0.01	0.0	1.02	0.81	1.02	1.02	0.75
3	25SL	0.35	0.47	0.62	0.61	0.77	0.0	0.03	-0.01	0.03	-0.07	0.00	0.0	1.15	1.11	1.15	0.85	0.54
4	25SR	0.44	0.70	0.62	0.89	0.87	0.0	-0.03	0.06	-0.11	0.09	0.00	0.0	1.04	4.37	1.17	1.13	0.62
5	15N	0.62	0.80	0.86	0.90	0.91	0.0	-0.01	0.01	0.02	0.01	-0.02	0.0	1.19	0.32	1.15	0.99	0.53
6	15SL	0.68	0.82	0.84	0.85	0.93	0.0	0.04	0.03	-0.01	-0.04	-0.00	0.0	1.38	1.20	1.36	0.80	0.43
7	15SR	0.85	0.84	0.94	0.97	1.00	0.0	0.03	-0.07	0.09	0.01	0.03	0.0	1.02	3.28	1.11	0.52	0.44
8	7C	0.90	0.91	0.95	0.94	0.99	0.0	0.04	-0.02	-0.00	-0.02	0.01	0.0	1.13	1.71	1.16	0.81	0.31
SCORE RANGE		5-30	31-35	36-38	39-41	42-48	0- 0	N= 26	23	25	25	27	0	121	5	126	DEG OF FREM	
MEAN ABILITY		0.12	0.66	1.26	1.66	2.12	0.0							0.13	0.63	0.13	STD ERROR.	
GROUP MN SQ		1.2	1.9	2.5	2.6	1.0	0.3											
SD(MN SQ)		0.9	3.1	3.9	2.6	1.9	0.0											
EXPCTD SC		0.8	0.8	0.5	0.8	0.8												

FLLS=TOO MANY RIGHT
WINLS=TOO MANY WRONG

MILITARY POLICE DATA--HITS PER TARGET

SERIAL ORDER / DIFFICULTY ORDER / FIT ORDER

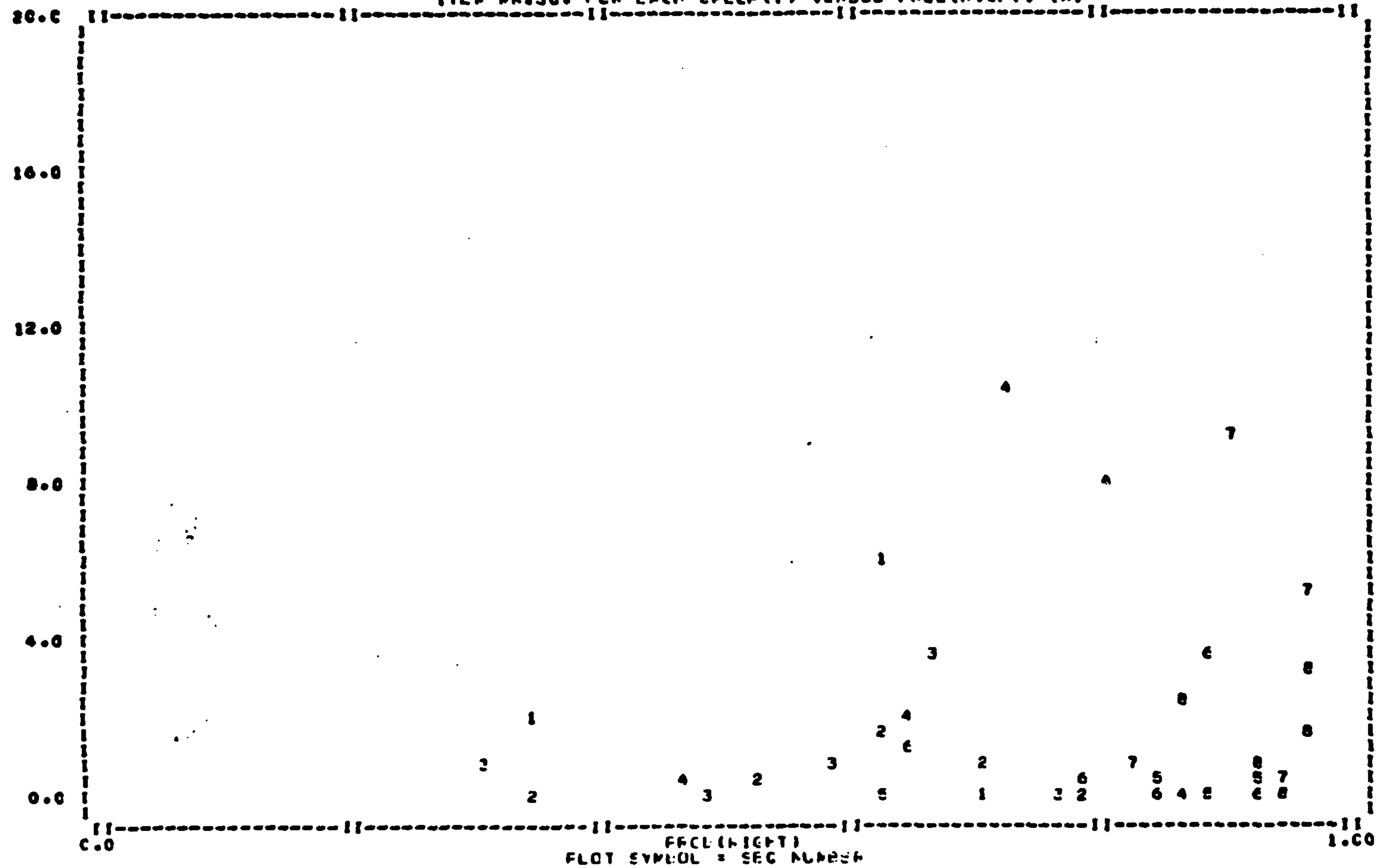
SEQ NUM	ITEM NAME	ITEM DIFF	DISC INDX	FIT MN SQ	SEQ NUM	ITEM NAME	ITEM DIFF	DISC INDX	FIT MN SQ	SEQ NUM	ITEM NAME	ITEM DIFF	DISC INDX	FIT MN SQ	POINT BISER
1	35P	0.74	1.09	1.53	8	7C	-1.77	0.81	1.15	2	25N	0.75	1.02	1.01	0.75
2	25N	0.75	1.02	1.01	7	15SR	-1.48	0.52	1.10	7	15SR	-1.48	0.52	1.10	0.44
3	25SL	0.53	0.99	1.15	6	15SL	-0.46	0.80	1.37	5	15N	-0.44	0.99	1.15	0.53
4	25SR	0.26	1.13	1.17	5	15N	-0.44	0.99	1.15	3	25SL	0.53	0.85	1.15	0.54
5	15N	-0.44	0.99	1.15	4	25SR	0.26	1.13	1.17	8	7C	-1.77	0.81	1.15	0.31
6	15SL	-0.46	0.80	1.37	1	35P	0.74	1.09	1.53	4	25SR	0.26	1.13	1.17	0.62
7	15SR	-1.48	0.52	1.10	2	25N	0.75	1.02	1.01	6	15SL	-0.46	0.80	1.37	0.43
8	7C	-1.77	0.81	1.15	3	25SL	0.53	0.85	1.15	1	35P	0.74	1.09	1.53	0.65
	MEAN	0.00	0.98	1.22											
	S.D.	0.55	0.11	0.18											

CORRELATION DIFF-DISC= 0.62 DIFF-MNSQ= 0.15 DISC-MNSQ= 0.21

MILITARY PCLICE CATA--HITS PEF TARGET

FAGE 9

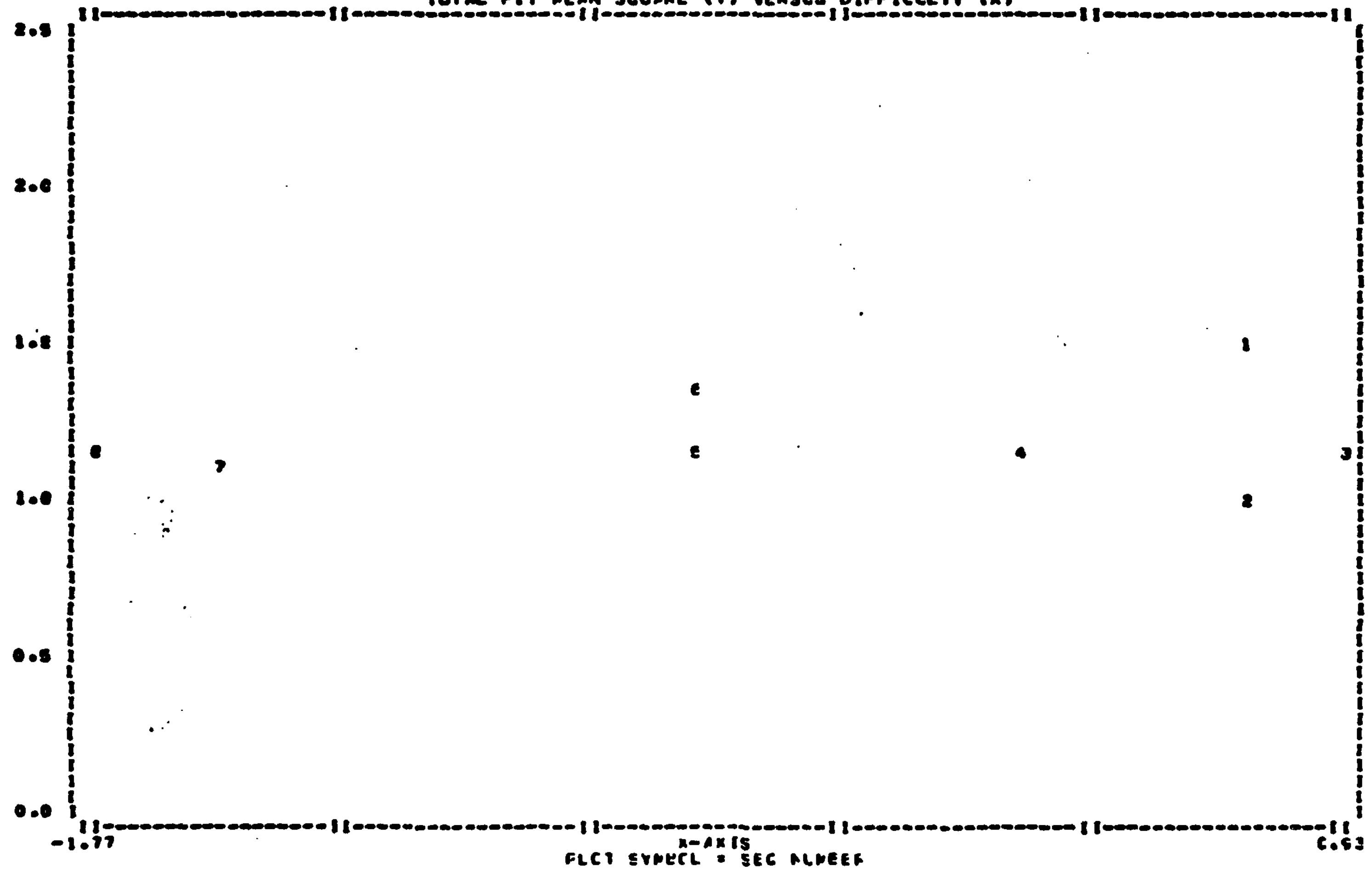
ITEM NA.SC. FCR EACH GFCLP(Y) VERSUS FRCE(RIGHT) (X)



MILITARY POLICE DATA--HITS PER TARGET

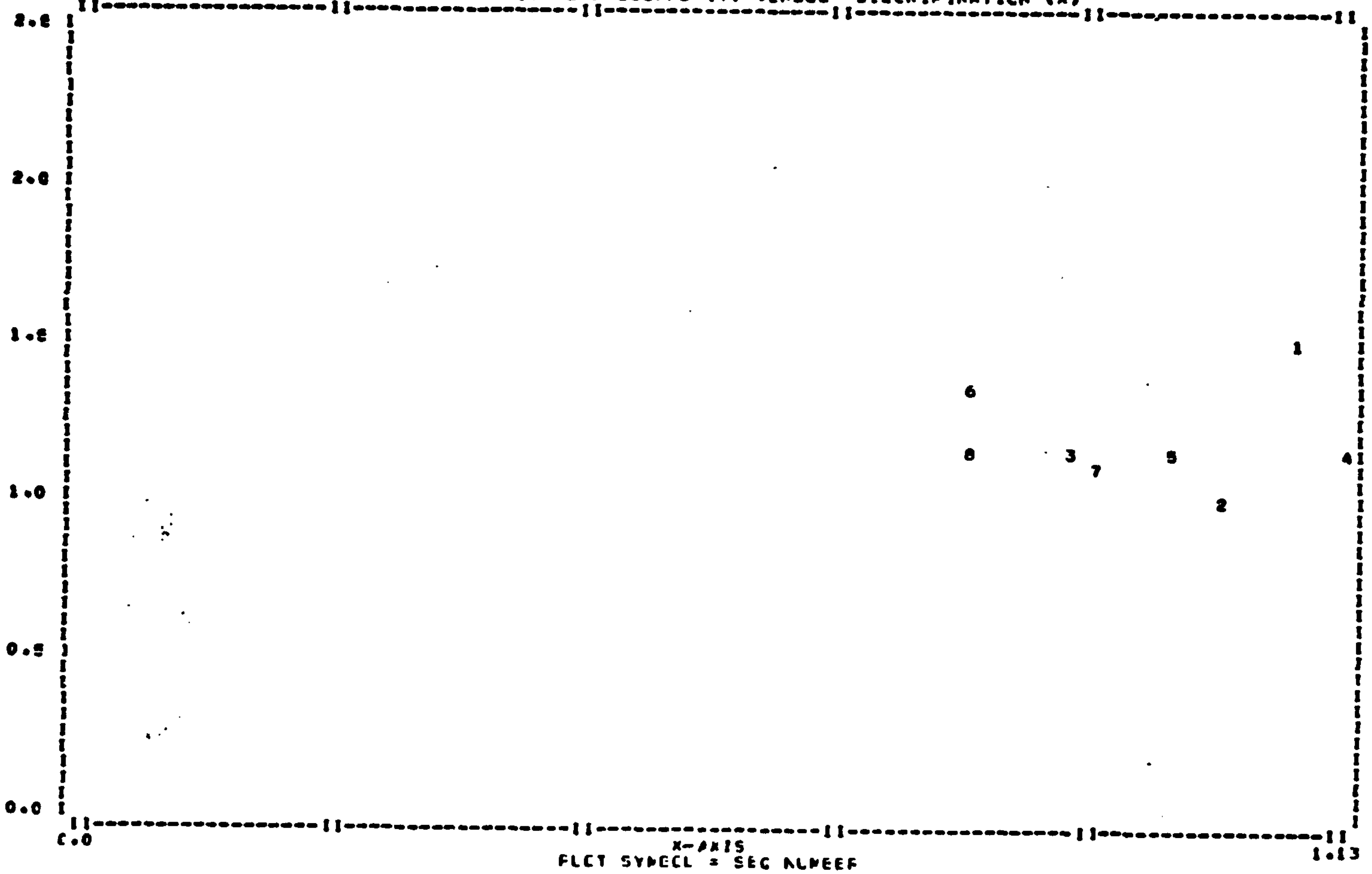
PAGE 10

TOTAL FIT MEAN SQUARE (Y) VERSUS DIFFICULTY (X)



MILITARY POLICE DATA--HITS PER TARGET

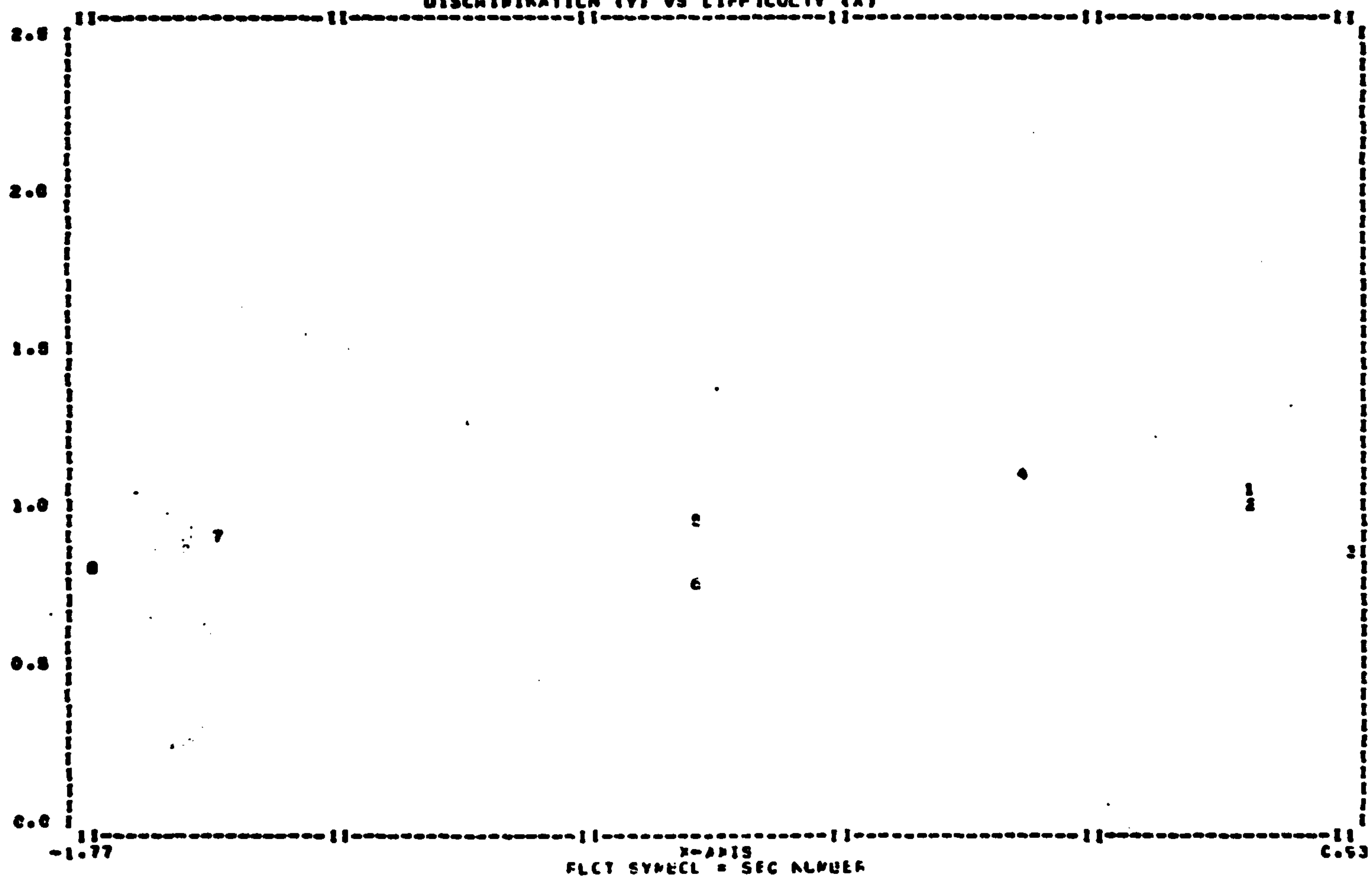
TOTAL FIT MEAN SQUARE (Y) VERSUS DISCRIMINATION (X)



MILITARY POLICE DATA--HITS PER TARGET

PAGE 12

DISCRIMINATION (Y) VS DIFFICULTY (X)



APPENDIX B

BICAL SOURCE PROGRAM LISTING

```
FORTRAN IV G1  RELEASE 2.0          MAIN             DATE = 76302          00/16/35

      DIMENSION MATX(1300),IB(1000),IS(150),DIFF(450),ABIL(1000)
      DIMENSION NSEL(1000),IDATA(150),SNAME(150),C(320),A(320),ID(150)
      DIMENSION ISEL(150),Z(150)
      COMMON  NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)
     1,SNAME
    1 CALL PAGE(1,I,J)
      CALL REDCP(IDATA,IB,IS,ISEL,MATX,A,C,DIFF)
      CALL EDITD(IB,IS,ISEL,MATX)
      CALL PAGE(2,I,J)
      WRITE(6,102)
      DO 4 I=1,LREC
    4 NSEL(I)=1
      IF(ISW(10).NE.1.AND.ISW(10).NE.3)CALL HSTGM(IB,LREC ,NSUBJ,0,NSEL)
      CALL PAGE(2,I,J)
      WRITE(6,103)
      CALL HSTGM(IS,NITEM,NSUBJ,1,ISEL)
      CALL ESTIM(IS,IB,DIFF,ABIL,ISEL,MATX,KCAB)
      CALL GFPIM(DIFF,Z,IB,ISEL,NSEL,MATX,IDATA,C,IS,ABIL,A,ID)
      CALL FITCS(Z,C,IB,ISEL,NSEL,DIFF,ABIL,IDATA,MATX,A)
      CALL SMMRY(DIFF,A    ,IDATA,IS,MATX,ISEL,C,IB)
      IF(ISW(10).GT.1) GO TO 1
      X=10.
      CALL GFLTR(ABIL,DIFF,IDATA,MATX, C,X)
      CALL PAGE(2,I,J)
      WRITE(6,100)
      X=2.5
      CALL FFLTR(DIFF,IDATA,IB,X)
      CALL PAGE(2,I,J)
      WRITE(6,104)
      CALL FFLTR(A,IDATA,IB,X)
      CALL PAGE(2,I,J)
      WRITE(6,101)
      DO 3 I=1,NITEM
      IF(A(I))2,3,3
    2 A(I)=0.0
    3 MATX(I)=100.0*A(I)
      X=2.5
      CALL FFLTR(DIFF,IDATA,MATX,X)
      GO TO 1
  100 FORMAT(40X,47HTOTAL FIT MEAN SQUARE (Y) VERSUS DIFFICULTY (X))
  101 FORMAT(40X,'DISCRIMINATION (Y) VS DIFFICULTY (X)')
  102 FORMAT(' SCORE',8X,'DISTRIBUTION OF ABILITY')
  103 FORMAT(' ITEM ',8X,'DISTRIBUTION OF EASINESS')
  104 FORMAT(40X,'TOTAL FIT MEAN SQUARE (Y) VERSUS  DISCRIMINATION (X)')
      END
```

```
FORTRAN IV G1  RELEASE 2.0              ABLTY               DATE = 76302          00/16/35
0001            SUBROUTINE ABLTY (AB,D,SE,IB,ISEL)
0002            DIMENSION AB(1),D(1),SE(1),ISEL(1),IE(1)
0003            COMMON  NITEM,NGRCP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)
               1,SNAME(1)
0004            DATA BLK/'  '/,AST/'*'/
0005            LLINE=0
0006            L=NITEM
0007            J=NITEM+LREC
          C**CHANGE TO EXPONENTIAL SCALE
0008        210 CONTINUE
0009            L1=LREC-1
          C**BEGON LOOP ON SCORE GROUPS
0010            DO 214 K=1,L1
          C**BEGIN ITERATION LOOP
0011            DO 215 ITK=1,5
0012            IX=K+L
0013            SE(IX )=0.0
0014            DD=0.0
          C**BEGIN LOOP OVER ITEMS
0015            DO 216 I=1,L
0016            IF(ISEL(I))216,216,213
0017        213 P=EXP(AB(K)-D(I))
0018            P=P/(1.0+P)
          C**COMPUTE SUM OF P AND PQ
0019            SE(IX )=SE(IX )+P*(1.0-P)*ISEL(I)
0020            DD=DD+P*ISEL(I)
0021        216 CONTINUE
0022            DD=(K-DD)/SE(IX)
0023            AB(K)=AB(K)+DD
          C**CHECK OFF CONVERGENGCE
0024            IF(ABS(DD)-0.05)214,215,215
0025        215 CONTINUE
0026        214 CONTINUE
0027            DO 9198 I=1,L1
          C**FINAL ABILITIES AND STANDARD ERRORS
0028            IX=I+L
0029       9198 SE(IX)=1.0/SQRT(SE(IX))
          C**CHANGE TO LOG SCALE
0030            DO 9200 I=1,L
0031            IF(ISEL(I))9200,9200,9199
0032       9199 IX=I +L+LREC
0033            SE(I)=1.0/SQRT(SE(I))
0034       9200 CONTINUE
          C**PRINT ABILITY TABLE
0035            WRITE(6,200)
0036        200 FORMAT('0',74('-')/1H ,   8HSEQUENCE,4H I   ,4HITEM,4H I   ,3X
               1'ITEM',6X,'STANDARD',3X9HLAST DIFF,3X,'PROX ',4X,'FIRST',5H   II )
0037            WRITE(6,201)
```

FORTRAN IV G1 RELEASE 2.0 ABLTY DATE = 76302 00/16/35

```
  201 FORMAT(1H ,7H NUMBER,5H   I  ,4HNAME,4H I  ,10HDIFFICULTY,5HSHERROR
     1,3X10H  CHANGE   ,3X,'DIFF ',4X,'CYCLE',5H   II /1X,74('-'))
      ROOT=0.0
      DO 4 I=1,NITEM
      IF(ISEL(I)) 4, 4, 5
    5  XK=XK - XINC
      K = XK
      IX=K+NITEM
      IY = I + NITEM + LREC
      X =          SE(IY)
      ROOT = ROOT + X*X
      IXX = I + NITEM
      IZZ = IXX + NITEM
      Y = C(IZZ)
      WRITE(6,101)I,SNAME(I),D(I),SE(I),X,C(IXX),Y
  101 FORMAT(1H ,I6,2X4H I   ,A4,3H I  ,F8.3,4XF9.3,2X3F9.3,2X3H II)
    4 CONTINUE
    9  ROOT=ROOT/(  IC -1)
       ROOT=SQRT(ROOT)
      INS=0
      XTSS2=0.
      XTSS=0.
      DO 4000 NTS=1,L1
      XTSS=XTSS+IB(NTS)*AB(NTS)
      XTSS2=XTSS2+IB(NTS)*AB(NTS)*AB(NTS)
 4000 INS=INS+IB(NTS)
      XTMN=XTSS/INS
      XTSD=(XTSS2-(XTSS*XTSS/INS))/(INS-1)
      XTSD=SQRT(XTSD)
      WRITE(6,105) ROOT,XTMN,XTSD
  105 FORMAT(1X74('-')/25X18HROOT MEAN SQUARE =,F5.3//
     127X16HMEAN ABILITY   = F6.2/27X,'SD OF ABILITY = ',F6.2)
      WRITE(6,102)
  102 FORMAT('1 COMPLETE SCORE EQUIVALENCE TABLE'/1X34('-'),13('------+'
     1)/'      RAW',10X,'LOG    STANDARD II',20X,'TEST CHARACTERSTC CURVE'/
     23X'SCORE COUNT ABILITY  ERRORS   II'/1X31('-'),'II+',13('------+'))
      K=LREC - 1
      L1 = K
      DO 10 I=1,L1
      N=(AB(K)+6.0)*7.0
      IF(N.GT.84)N=84
      IX=K+NITEM
      WRITE(6,103)K,IB(K),AB(K),SE(IX),(ELK,J=1,N),AST
  103 FORMAT(I7,I5,2F9.2,'  II',85A1)
   10 K=K-1
      WRITE(6,104)(I,I=1,6),XTMN,XTSD
  104 FORMAT(1X31('-'),'II+',13('------+')/26X,'-6      -5      -4      -3
     1      -2      -1         0',6I7/' MEAN ABILITY   =',F5.2/' SD OF ABILITY
     2=',F5.2)
       RETURN
      END
```

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```
0001           SUBROUTINE EDITO(IB,IS,ISEL,MATX)
0002           DIMENSION IB(1),IS(1),ISEL(1)
0003           COMMON  NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)
              1,SNAME(1)
0004           NLOW=0
0005           NHIGH=0
0006           ITCK=0
0007           NCUT=0
0008           IX=0
0009           IF(NGROP.LE.6) NGROP=30
0010           L=LREC
0011           MQ=L-1
0012           IF(MAXSC-L)2,1,1
0013         1 MAXSC=MQ
         C**COUNT HIGH AND LOW SCORES
0014         2 DO88 I=1,MQ
0015           IF(I-MINSC)4,6,5
0016         4 NLOW=NLOW+IB(I)
0017           GO TO 88
0018         5 IF(I-MAXSC)6,6,7
0019         6 IX=IX+IB(I)
0020           GO TO 88
0021         7 NHIGH=NHIGH+IB(I)
0022        88 CONTINUE
0023           WRITE(6,100)MINSC,NLOW,MAXSC,NHIGH,IX,NSUBJ
0024           NSUBJ=IX
0025           WRITE(6,202)
0026       202 FORMAT(1H ,//15H REJECTED ITEMS,//7H  ITEM ,3X4HITEM,3X8HANSWERED/
              17H NUMBER,3X4HNAME,3X9HCORRECTLY/1X30('-'))
0027         3 IF(NSUBJ-NGROP)99,99,21
0028        21 IF(MAXSC-MINSC)99,99,22
         C**BEGIN EDIT LOOP
0029        22 DO 18 I=1,NITEM
0030           K=1
0031           IF(ISEL(I)) 18,18,8
0032         8 IF(IS(I))15,15,9
0033         9 IF(IS(I)-NSUBJ*ISEL(I))18 ,10,10
         C**CHECK FOR PERFECT SCORES
0034        10 WRITE(6,201)I,SNAME(I),IS(I)
0035       201 FORMAT(1H ,2XI2,5XA4,4XI4,3X10HHIGH SCORE)
0036           ITCK=ITCK+1
         C**ADJUST MINSC, MAXSC, AND IB ARRAY FOR REJECTED PEOPLE
0037           IF(MINSC-ISEL(I))13,13,11
0038        13 NSUBJ=NSUBJ-IB(1)
0039           NOUT=NCUT+IB(1)
0040           MINSC=1 + ISEL(I)
0041           K=2
0042           J2=ISEL(I)+1
```

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```
   11 DO 12J=J2,L
      J1=J-ISEL(I)
   12 IB(J1)=IB(J)
      MINSC=MINSC-ISEL(I)
      MAXSC=MAXSC-ISEL(I)
      L=L-ISEL(I)
      ISEL(I)=0
      GO TO (18,3),K
C**CHECK FOR ZERO SCORES
   15 WRITE(6,200)I,SNAME(I),IS(I)
  200 FORMAT(1H ,2XI2,5XA4,4XI4,3X5HLOW SCORE)
      ITCK=1+ITCK
      L=L-ISEL(I)
      IF(L-MAXSC)17,17,18
17    DO 171 J=L,MAXSC
      NSUBJ=NSUBJ-IB(J)
171   NOUT=NOUT+IB(J)
      NOUT=NOUT+IB(MAXSC)
      MAXSC=L-ISEL(I)
      ISEL(I)=0
      GO TO 3
   18 CONTINUE
C**PRINT RESULTS OF DATA EDITTING
  100 FORMAT(                    15H SUBJECTS BELOW,5XI3,6XI4/15H SUBJECTS A
     1BOVE,5XI3,6XI4/19H SUBJECTS IN CALIB.I14/1H ,32(1H-)/
     215H TOTAL SUBJECTS,13XI5)
  102 FORMAT(45HO PROBLEM ENDED BECAUSE OF 1 OF THE FOLLOWING/
     117H SUBJECTS READ IN I6/18H SUBJECTS IN RANGE I5/14H MAXIMUM SCORE
     2I5/14H MINIMUM SCORE I5)
      IF(ITCK)19,19,20
   19 WRITE(6,211)
  211 FORMAT(10X,'NONE')
   20 WRITE(6,210)NOUT,NSUBJ,ITCK,L,MINSC,MAXSC
  210 FORMAT(1H0,20H  SUBJECTS DELETED =,I5/21H SUBJECTS REMAINING =,I5/
     1/6X,'ITEMS DELETED =',I5/4X17HPOSSIBLE SCORE  =,I5//6X15HMINIMUM S
     2CORE =,I5/6X15HMAXIMUM SCORE =,I5)
      IC=IC-ITCK
      LREC=L
      RETURN
   99 WRITE(6,102) IC,NSUBJ,MAXSC,MINSC
      STOP
      END
```

75

```
FORTRAN IV G1  RELEASE 2.0            ESTIM              DATE = 76302          00/16/35

0001                SUBROUTINE ESTIM(IS,IB,DIFF,ABIL,ISEL,SE   ,M)
0002                DIMENSION IS(1),IB(1),DIFF(1),ABIL(1),ISEL(1),SE(1)
0003                DIMENSION PROC(3)
0004                COMMON  NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)
                   1,SNAME(1)
0005                DATA PROC/'PROX','UCON','EROR'/
0006                CALL PAGE(2,I,J)
0007                WRITE(6,199) PROC(M)
0008            199 FORMAT(1H0,15HPROCEDURE USED ,A4)
              C**NORMAL APPROXIAMATION METHOD
0009                CALL PROX(IS,IB,DIFF,ABIL,ISEL,SE,M)
0010                GO TO (3,1,1),M
              C**CORRECTED UNCONDITIONAL METHOD
0011              1 CALL UCON(IS,IB,DIFF,ABIL,ISEL,SE)
0012              3 CONTINUE
              C**COMPUTE FIANL ABILITIES AND PRINT TABLE
0013                CALL ABLTY(ABIL,DIFF,SE,IB,ISEL)
0014                RETURN
0015                END
```

```
FORTRAN IV G1   RELEASE 2.0              FITCS              DATE = 76302          00/16/35

0001              SUBROUTINE FITCS(ZZ,B,IB,ISEL,NSEL,DIFF,ABIL,IDATA,MATX,A)
0002              DIMENSION IB(1),ISEL(1),NSEL(1),DIFF(1),ABIL(1),IDATA(1),MATX(1)
0003              DIMENSION EXT(6),VAR(6),OBS(6),COMP(18),DISC(6),Z(6),STAT(2)
0004              DIMENSION ESD(4),ZZ(1),B(1),IGROP(6),A(1),TOT(6),SE(6)
0005              COMMON NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)
                 1,SNAME(1)
0006              CALL PAGE(2,I,J)
            C**CLEAR ARRAYS
0007              LLINE=0
0008              CRIT=0.1
0009              DO 197 I=1,6
0010              Z(I)=0.0
0011              DISC(I)=0.0
0012              OBS(I)=0.0
0013              TOT(I)=0.0
0014              SE(I)=0.0
0015          197 IGROP(I)=0
            C**COUNT NUMBER IN EACH SCORE GROUP
0016              DO 198 I=MINSC,MAXSC
0017              J=NSEL(I)
0018              IF(J)198,198,196
0019          196 IGROP(J)=IGROP(J)+IB(I)
0020          198 CONTINUE
0021              DO 195 I=1,18
0022          195 COMP(I)=0.0
0023              WRITE(6,200)
0024              WRITE(6,201)
0025              WRITE(6,202)
0026          200 FORMAT(1H ,18X25HITEM CHARACTERISTIC CURVE11X27HDEPARTURE FROM EXP
                 1ECTED LOC,18X16HFIT MEAN SQUARE /1X129('-'))
0027          201 FORMAT(1H ,12H SEQ  ITEM I,2(38H  1ST   2ND   3RD   4TH   5TH   6T
                 1H  I) ,'  WITHN   BETWN        DISC  POINT  I')
0028          202 FORMAT(1H ,12H NUM  NAME I,6(6H GROUP),2H I,6(6H GROUP),2H I,
                 1'  GROUP   GROUP  TOTAL    INDX  BISER  I'/1X129('-'))
            C**COMPUTE AVERAGE ABILITY FOR EACH SCORE GROUP
0029              X=0.0
0030              DO 181 I=MINSC,MAXSC
0031              J=NSEL(I)
0032              IF(J)181,181,182
0033          182 COMP(J)=COMP(J)+IB(I)*ABIL(I)
0034              X=X+IB(I)*ABIL(I)
0035          181 CONTINUE
0036              X=X/NSUBJ
            C**STORE AVERAGE IN LOC SEVEN TO TWELVE AND CENTER LOC ONE TO SIX AT ZER
0037              DO 184 I=1,NGROP
0038              COMP(I)=COMP(I)/IGROP(I)
0039              COMP(I+6)=COMP(I)
0040          184 COMP(I)=COMP(I)-X
```

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```
FORTRAN IV G1  RELEASE 2.0                FITCS                DATE = 76302          00/16/35

0041                NI=LREC-1
              C**CHANGE SCALE
              C**BEGIN LOOP THROUGH ITEMS
0042                DO 9 K=1,NITEM
0043                L=IDATA(K)
0044                IF(ISEL(L))9,9,1
0045              1 LL=L
0046                LK=L
0047                X=DIFF(L)
0048                DO 11 I=1,NGROP
0049                EXT(I)=0.0
0050             11 VAR(I)=0.0
              C**COMPUTE EXPECTED NUMBER NP AND VARIANCE NPQ
0051                DO 8 I=MINSC,MAXSC
0052                J=NSEL(I)
0053                IF(J)8,8,7
0054              7 P=EXP(ABIL(I)-X)
0055                P=P/(1.0+P)
0056                EXT(J)=EXT(J)+IB(I)*P*ISEL(L)
0057                VAR(J)=VAR(J)+IB(I)*P*(1.-P)*ISEL(L)
0058              8 CONTINUE
0059                P=0.0
0060                STAT(1)=0.0
0061                STAT(2)=0.0
              C**BEGIN LOOP THROUGH SCORE GROUPS
0062                DO 2 I=1,NGROP
              C**OBSERVED NUMBER OF CORRECT RESPONSES
0063                ISC=MATX(LL)
0064                SC=ISC
0065                VI=SQRT(VAR(I))
              C**CHECK FOR ZERO OR PERFECT SCORES WITHIN A GROUP
0066                IF(ISC)3,3,4
0067              4 IF(ISC-IGROP(I)*ISEL(L))6,3,3
0068              3 VAR(I)=0.0
0069                DISC(I)=99.0
0070                GO TO 12
0071              6 DD=X
              C**COMPUTE DIFFICULTY ESTIMATE BASED ON THE ONE GROUP
0072                CALL NEWT(DD,ISEL(L),ABIL,IB,ISC,NSEL,MINSC,MAXSC,I,CRIT,6,XX,Y)
0073                DISC(I)=DD-DIFF(L)
              C**STANDARDIZE THE DEVIATION
0074             12 Z(I)=(SC-EXT(I))/VI
              C**SCALE TO A SINGLE DEGREE OF FREEDOM
0075                XX=IC
0076                XX=XX/(XX-1.0)
0077                Y=NSUBJ-IGROP(I)
0078                XX=XX*NSUBJ/Y
0079                Z(I)=Z(I)*Z(I)*XX
```

```
FORTRAN IV G1   RELEASE 2.0            FITCS              DATE = 76302          00/16/35
0080              XX=Z(I)
          C**STORE Z SQUARE AS AN ITEGER FOR GPLTR
0081              MATX(LL)=100.0*XX
0082              LL=LL+NITEM
          C**COMPUTE MARGINALS OF FIT TABLE
0083              TCT(I)=TCT(I)+XX
0084              SE(I)=SE(I)+XX*XX
0085              P=P+Z(I)
0086              DES(I)= SC/(IGROP(I)*ISEL(L))
          C**COMPUTE DISCRIMIANTION INDEX FOR TIS ITME
0087              XX=COMF(I)
0088              STAT(1)=STAT(1)+XX*DISC(I)*VAR(I)
0089              DISC(I)=(SC-EXT(I))/(IGROP(I)*ISEL(L))
0090            2 STAT(2)=STAT(2)+XX*XX*VAR(I)
0091              STAT(1)=-STAT(1)/STAT(2) +1.0
0092              X = ZZ(LK)*NSUBJ - P
0093              X = X/(NSUBJ-NGROP)
0094              P = P/NGROP
          C**PRINT LINE OF THE FIT TABLE FOR THIS ITEM
0095              CALL PAGE(4,2,LLINE)
0096              WRITE(6,203)LK,SNAME(LK),OBS,DISC,X,F,ZZ(LK),STAT(1),B(LK)
0097          203 FORMAT(1H ,I4,2X,A4,2H I,6(F6.2),2H I,6(F6.2),2H I,
                 13F7.2,1X,2F7.2,'  I')
          C**STORE DISCRIMINATION INDEX
0098              A(L)=STAT(1)
0099            9 CONTINUE
0100              DO 5 I=1,NITEM
0101              IF(ISEL(I).LE.0) GO TO 5
0102              IB(I) = 100 * ZZ(I)
0103          5   CONTINUE
          C**COMPUTE STANDARD DEVIATION OF Z SQUARES
0104              DO 17 I=1,NGROP
0105              TCT(I)=TCT(I)/IC
0106           17 SE(I)=SQRT((SE(I)- IC *TCT(I)*TCT(I))/( IC -1))
          C**STORE AVERAGE ABILITY FOR GPLTR
0107              DO 10 I=1,NGROP
0108              J=I+6
0109           10 ABIL(I)=COMP(J)
0110              DO 15 I=2,12
0111           15 IDATA(I)=0
          C**CALCULATE LIMITS OF EACH SCORE GROUP
0112              DO 14 I=1,LREC
0113              J=NSEL(I)+2
0114              IF(J)14,14,13
0115           13 IDATA(J)=I
0116           14 CONTINUE
0117              IDATA(1)=MINSC
0118              J=2*NGROP
```

```
0119          IDATA(J)=MAXSC
0120          J=J-1
0121          DO 16 I=3,J,2
0122       16 IDATA(I)=IDATA(I-1)+1
        C**PRINT OUTPUT LINES
0123          ESD(1) = SQRT(2.0/(NSUBJ-NGRCP))
0124          ESD(2) = SQRT(2.0/NGRCP)
0125          ESD(3) = SQRT(2.0/NSUBJ)
0126          ESD(4) = SQRT(2.0/IC)
0127          K = NSUBJ - NGROP
0128          WRITE(6,204)(IDATA(I),I=1,12),(IGRCP(I),I=1,6),K,NGRCP,NSUBJ,
             1(COMF(I),I=7,12),(ESD(I),I=1,3),TOT,SE,(ESD(4),I=1,NGROP)
0129      204 FORMAT(1H ,129('-')/     ' SCORE RANGE ',6(I3,'-',I2),'  N=',I4,516,
             12X3I7,'  DEG OF FREDM'/13H MEAN ABILITY,6F6.2,5X,'PLUS=TOO MANY RIG
             2HT'/58X,
             2'MINUS=TOO MANY WRONG',11X3F7.2,'  STD ERROR'/' GROUP MN SQ',6F6.1
             3/' SD(MN SQ)   ',6F6.1/' EXPCTD SD   ',6F6.1)
0130          RETURN
0131          END
```

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```
0001      SUBROUTINE FFLTR     (DIFF,IDATA,MATX,X2)
0002      DIMENSION ICHAR(11),           DIFF(1),IDATA(1),MATX(1)
0003      COMMON   NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)
         1,SNAME(1)
0004      DATA  ICHAR/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,1H0,1H /,IBLK/2H  /
0005      X1=X2
0006      DMAX=0.0
0007      DMIN=0.0
    C**FIND LIMITS OF X AXIS
0008      DO 4 I=1,NITEM
0009      IF(DIFF(I)-DMIN)1,2,2
0010    1 DMIN=DIFF(I)
0011      GO TO 4
0012    2 IF(DIFF(I)-DMAX)4,4,3
0013    3 DMAX=DIFF(I)
0014    4 CONTINUE
    C**COMPUTE SCALE FACTOR
0015      DMAX=(DMAX-DMIN)/ 50.0
    C**SORT INTO ORDER DEFINED BY CONTENTS OF MATX
0016      CALL SRT(MATX,IDATA)
0017      NIT=NITEM
0018      K=IDATA(NITEM)
    C**COMPUTE SCALE UNIT FOR Y AXIS
0019      UNIT=MATX(K)/100.
0020      IF(UNIT-X1)21,21,20
0021   20 X1=X1*2.0
0022   21 UNIT=X1
0023      UNIT=UNIT/50.0
0024      WRITE(6,102)
0025      K=2*(NITEM)+102
0026      KK=2*NITEM+1
0027      LINE=51
    C**PRINT LABEL FOR Y AXIS
0028   22 X11=X1+.01
    C**PRINT TEN LINES
0029      DO 14 I=1,10
0030      DO 98 J=KK,K
0031   98 MATX(J)=IBLK
0032 97   IF(NIT)13,13,99
0033   99 K=IDATA(NIT)
0034      X    =MATX(K)/100.
    C**DOES MATX K GO ON TIS LINE
0035      IF(X-X1)13,5,5
0036    5 X=DIFF(K)-DMIN
0037      X=X/DMAX
    C**FIND CORRECT COLUMN
0038      J=X
0039      J=2*(J+NITEM)+1
```

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```
C**LOCATE SYMBOLS FOR SEQ NUMBER
      KK=K/10
      IF(KK)7,7,8
    7 KK=11
    8 MATX(J)=ICHAR(KK)
      IF(KK-10)10,9,9
    9 KK=0
   10 KK=K-KK*10
      IF(KK)11,11,12
   11 KK=10
   12 J=J+1
      MATX(J)=ICHAR(KK)
      NIT=NIT-1
      GO TO 97
   13 X1=X1-UNIT
      KK=2*NITEM+1
      K=2*NITEM+102
C**PRINT LINE WHEN ALL ITEMS HAVE BEEN CHECKED
      IF(I.EQ.1) GO TO 23
      WRITE(6,101)(MATX(J),J=KK,K)
      GO TO 24
23    WRITE(6,104)X11,(MATX(J),J=KK,K)
24    LINE=LINE-1
      IF(LINE)15,15,14
   14 CONTINUE
      GO TO 22
   15 CONTINUE
C**PRINT BOTTOM LINES
      DMAX=DMIN+ 50.0*DMAX
      WRITE(6,102)
      WRITE(6,103) DMIN,DMAX
  101 FORMAT(1H 10X1HI,102A1,1HI)
  102 FORMAT(1H , 11X,5('II------------------'),'II')
  103 FORMAT(10XF5.2,43X,'  X-AXIS  ',42XF5.2/
     151X24HPLOT SYMBOL = SEQ NUMBER)
  104 FORMAT(1X,F9.1,' I',102A1,'I')
      RETURN
      END
```

```
FORTRAN IV G1  RELEASE 2.0            GPLTR               DATE = 76302          00/16/35

0001            SUBROUTINE GPLTR(ABIL,DIFF,ICATA,MATX,B,XX)
0002            DIMENSION ICHAR(11),ABIL(1),DIFF(1),ICATA(1),MATX(1),LINE(102),B(1)
0003            COMMON  NITEM,NGROUP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)
               1,SNAME(1)
0004            DATA ICHAR/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,1H0,1H /,IBLK/2H  /
0005            CALL PAGE(2,I,J)
0006            WRITE(6,100)
0007            WRITE(6,102)
0008            DMAX=1.0/50.0
0009            DMIN=0.0
0010            KK=0
0011            K=1
          C**LOCATE MAXIMUM
0012            DO 6 I=1,NITEM
0013            B(I)=0.0
0014            DO 6 J=1,NGROP
0015            IF(KK-MATX(K))5,6,6
0016          5 KK=MATX(K)
0017          6 K=K+1
0018            X=FLOAT(KK)/100.0
          C**COMPUTE SCALE FACTORS
0019            IF(X-XX)2,2,1
0020          1 XX=2.0*XX
0021          2 XX=XX/50.0
0022            K=1
0023            DO 7 J=1,NGROP
0024            DO 7 I=1,NITEM
0025            X=MATX(K)/100.
0026            B(I)=B(I)+X
0027            KK=X/XX+1.
0028            IF(KK-50)23,23,22
0029         22 KK=50
0030         23 MATX(K)=KK
0031          7 K=K+1
0032            NLNE=50
          C**PRINT LABELS FOR Y AXIS
0033          4 X1=(NLNE*XX)
0034            WRITE(6,104)X1
          C**PRINT TEN LINES
0035            DO 17ILL=1,10
0036            DO 8 I=1,102
0037          8 LINE(I)=IBLK
0038            K=1
          C**LOOP THROUGH GROUPS AND ITEMS
0039            DO 16 J=1,NGROP
0040            DD=ABIL(J)-DMIN
0041            DO 16 I=1,NITEM
0042            KK=MATX(K)
```

```
FORTRAN IV G1  RELEASE 2.0           GFLTR                CATE = 76302          00/16/35
                C**IS THIS THE RIGHT LINE
 0043                 IF(KK-NLNE)16,9,16
 0044               9 X=DD-DIFF(I)
                C**COMPUTE PROBAUILITY CF SUCESS
 0045                 X=EXP(X)
 0046                 X=X/(1.+X)
 0047                 X=X/DMAX
 0048                 KK=X
 0049                 KK=2*KK+1
                C**LOCATE SYMUCL FOR SEQ NUMBER
 0050                 ISYM=I/10
 0051                 IF(ISYM)10,10,11
 0052              10 ISYM=11
 0053              11 LINE(KK)=ICHAR(ISYM)
 0054                 IF(ISYM-10)13,12,12
 0055              12 ISYM=0
 0056              13 ISYM=I-ISYM*10
 0057                 IF(ISYM)14,14,15
 0058              14 ISYM=10
 0059              15 KK=KK+1
 0060                 LINE(KK)=ICHAR(ISYM)
 0061              16 K=K+1
                C**PRINT LINE WHEN FILLED
 0062                 WRITE(6,101) LINE
 0063                 IF(NLNE)3,3,17
 0064              17 NLNE=NLNE-1
 0065                 GO TO 4
                C**COMPUTE ITEM MEAN SQUARE FOR SUMMRY AND FFLTR
 0066               3 DO 18 I=1,NITEM
 0067              18 MATX(I)=100.0*B(I)/NGRCP
 0068                 DMAX=1.0
                C**PRINT BOTTOM LINES
 0069                 WRITE(6,102)
 0070                 WRITE(6,103) DMIN,DMAX
 0071             100 FORMAT(40X,'ITEM MN.SQ. FOR EACH GRCUP(Y) VERSUS PRCB(RIGHT) (X)')
 0072             101 FORMAT(1H 10X1HI,102A1,1HI)
 0073             102 FORMAT(1H , 11X,5('II-----------------'),'II')
 0074             103 FORMAT(10XF5.2,43X,'PRCB(RIGHT)',41XF5.2/
                     150X24HPLOT SYMBOL = SEQ NUMBER)
 0075             104 FORMAT(1H+,F9.1)
 0076                 RETURN
 0077                 END
```

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FORTRAN IV G1 RELEASE 2.0 GRPIM DATE = 76302 00/16/35

```
      SUBROUTINE GRPIM(D,Z,IB,ISEL,NSEL,MATX,ICATA,B,IS,AB,A,ID)
      DIMENSION NSEL(1),IB(1),ISEL(1),MATX(1),ICATA(1),E(1),IS(1),AB(1)
      DIMENSION D(1),Z(1),ID(1),A(1)
      COMMON NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)
     1,SNAME(1)
C**DETERMINE NUMBER OF SCORE GROUPS
   24 X=NSUBJ/NGROP
      NGROP=X
      IF(NGROP-6)22,22,21
   21 NGROP=6
22    X=NSUBJ/NGROP
      IX=NGROP*NITEM
      DO 1 I=1,IX
    1 MATX(I)=0
      DO 25 I=1,NITEM
      Z(I)=0.0
      A(I)=0.0
   25 B(I)=0.0
      SST=0.0
      SSTT=0.0
      DO 2 I=1,MINSC
    2 NSEL(I)=0
      DO 3 I=MAXSC,LREC
    3 NSEL(I)=0
      NSC=0
      IX=X
      L=1
C**DETERMINE GROUPINGS OF SCORES AS EQUAL AS POSSIBLE
      DO 8 I=MINSC,MAXSC
      NSC=NSC+IB(I)
      J=(NSC-IX)*2
      IF(J)7,5,4
    4 IF(J-IB(I))5,5,6
    5 NSEL(I)=L
      L=L+1
      NSC=0
      IF(L-NGROP)8,29,29
    6 L=L+1
      IF(L-NGROP)20,30,30
   20 NSC=IB(I)
    7 NSEL(I)=L
    8 CONTINUE
      GO TO 32
   29 I=I+1
   30 DO 31 L=I,MAXSC
   31 NSEL(L)=NGROP
C**READ AND SCORE SCRATCH FILE
32    ISWT=1
```

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```
            LLIM=ISW(2)
            KLIM=ISW(3)
            IFILE=ISW(4)
            IF(IFILE)16,40,15
      15    IFILE=-IFILE
            ISWT=2
            GO TO 40
      16    ISWT=3
       40   NSC=0
            READ(1,END=12) (IDATA(N),N=1,NITEM),(ID(N),N=LLIM,KLIM)
            DO 162 N=1,NITEM
            IF(ISEL(N))162,162,161
      161   NSC=NSC+IDATA(N)
      162   CONTINUE
            GO TO (17,9,10),ISWT
      9     WRITE(IFILE,100)NSC,AB(NSC),(ID(N),N=LLIM,KLIM)
      100   FORMAT(I4,F6.2,70A1/(80A1))
            GO TO 17
      10    WRITE(IFILE)NSC,AB(NSC),(ID(N),N=LLIM,KLIM)
      17    SC=NSC
            K=NSEL(NSC)-1
            IF(K)40,11,11
         11 K=K*NITEM
C**FILL MATX ARRAYS--SCORE GROUPS BY ITEMS
            ABIL = AB(NSC)
            DO 14 N=1,NITEM
            K=K+1
            IF(ISEL(N))14,14,141
      141   MATX(K)=MATX(K)+IDATA(N)
C**COMPUTE POINT BISERIALS
            B(N)=B(N)+ SC*IDATA(N)
            A(N)=A(N)+IDATA(N)*IDATA(N)
            P=EXP(ABIL-D(N))
            F=P/(1.+F)
            X=IDATA(N)-P*ISEL(N)
            Z(N)=Z(N)+X*X/((ISEL(N)-ISEL(N)*F)*F)
         14 CONTINUE
            SST=SST+SC
            SSTT=SSTT+SC*SC
            GO TO 40
         12 CONTINUE
            X=SST*SST/NSUBJ
            X=SSTT-X
            DO 13 K=1,NITEM
            IF(ISEL(K))13,13,131
      131   XY=IS(K)*SST/NSUBJ
            Y=IS(K)*IS(K)/NSUBJ
            XY= B(K)-XY
            Y= A(K)-Y
            B(K)=XY/SQRT(X*Y)
            Z(K) = Z(K)/(NSUBJ-1)
C**SET ORDER FOR PRINTING FIT TABLE--IN SEQUENCE
         13 IDATA(K)=K
            REWIND 1
            RETURN
            END
```

```
FORTRAN IV G1  RELEASE 2.0              HSTGM              DATE = 76302           00/16/35
0001            SUBROUTINE HSTGM(IS,K,L,IP,ITLE)
0002            DIMENSION IS(1),ITLE(1)
0003            DATA IX/'X'/,II/1HI/
0004            M=L
0005            XL=L
          C**CHECK METHOD FOR STTING SCALE FACTOR
0006            IF(IP)11,11,14
0007         11 M=IS(1)
          C**FIND LARGEST ENTRY
0008            DO 13 I=2,K
0009            IF(M-IS(I))12,13,13
0010         12 M=IS(I)
0011         13 CONTINUE
0012            IF(M.LT.50) M= 50
0013         14 XLL=M/XL
          C**PRINT HEADING
0014            WRITE(6,102)(J,J=2,10,2),II,II
0015            DO 4 I=1,K
          C**CHECK IF LINE IS TO BE INCLUDED
0016            IF(ITLE(I))4,4,2
0017          2 XIS=IS(I)
0018            PROP=XIS/(XL*ITLE(I))
          C**PRINT EDGES OF TABLE
0019            WRITE(6,100)I,      IS(I),PROP,II,II
0020            J=(50.*PROP)/XLL + .5
0021            IF(J-1) 4,4,3
          C**PRINT HISTOGRAM
0022          3 WRITE(6,101) (IX,IJ=1,J)
0023          4 CONTINUE
          C**PRINT BOTTOM LINE
0024            XLL = 2.00 * XLL
0025            WRITE(6,104) XLL
0026        100 FORMAT(I5,I6,4X   ,F6.2,2XA1,50XA1)
0027        101 FORMAT(1H+,23X50A1)
0028        102 FORMAT(          1H0,75(1H-)/1H ,6X5HCOUNT,2X11HPROPORTION ,5I10/
           123XA1,1H*,9(1H*),1H0,4(10H*********0),A1)
0029        104 FORMAT(1H ,75(1H-)/30X9H EACH X =,F5.2,' PERCENT')
0030            RETURN
0031            END
```

```
FORTRAN IV G1  RELEASE 2.0            PAGE              DATE = 76302          00/16/35

0001                SUBROUTINE PAGE(M,K,L)
0002                DIMENSION TITL(20)
0003                COMMON  NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)
                   1,SNAME(1)
0004                DATA A/'****'/
0005                GO TO (1,2,5,6),M
0006              1 READ(5,102) TITL
0007                IF(TITL (1)-A)3,4,3
0008              3 READ(5,104) NITEM,NGROP,MINSC,MAXSC,LREC,KCAB,NSUBJ,ISW
0009                READ(5,102)(SNAME(N),N=1,NITEM)
0010                I=1
0011                WRITE(6,101) TITL,I
0012                WRITE(6,100)(K,K=1,11),NITEM,NGROP,MINSC,MAXSC,LREC,KCAB,NSUBJ,ISW
0013                ISW(8)=NSUBJ
0014                ISW(10)=ISW(5)
0015                ISW(11) = ISW(6)
0016                IS=ISW(4)
0017                IF(ISW(2).EQ.0)ISW(2)=1
0018                IF(ISW(4).EQ.0)RETURN
0019                GO TO 5
0020              2 I=I+1
0021                WRITE(6,101) TITL,I
0022                RETURN
0023              4 CALL EXIT
0024           5    IF(IS.EQ.0)IS=7
0025                WRITE(IS,102) TITL
0026                RETURN
0027              6 IF(L-50)21,22,22
0028             21 L=L+1
0029                RETURN
0030             22 L=0
0031                I=I+1
0032                WRITE(6,103)
0033                WRITE(6,101) TITL,I
0034                GO TO (31,32,33),K
0035             31 WRITE(6,131)
0036                WRITE(6,231)
0037                RETURN
0038             32 WRITE(6,132)
0039                WRITE(6,232)
0040                WRITE(6,332)
0041                RETURN
0042             33 WRITE(6,133)
0043                RETURN
0044          100   FORMAT('0 CONTROL PARAMETERS'/' NITEM NGROP MINSC MAXSC  LREC  KCA
                   1B SCORE',11I6/18I6)
0045            101 FORMAT(1H1,20A4,40X,  4HPAGE,I3)
0046            102 FORMAT(20A4)
```

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```
FCRTRAN IV G1  RELEASE 2.0              PAGE              DATE = 76302          00/16/35
0047      103 FCRMAT(1CX,11C('*')/53X,'TABLE CONTINUED')                                    812.
0C48      104 FORMAT(14I5,4I1)                                                              813.
0049      131 FORMAT(1H ,   8HSEQUENCE,4H I   ,4HITEM,4H I   ,3X4HITEM,6X8HSTANDAR 814.
             1D,3X9HLAST DIFF,4H I   ,5HSCCRE,4X5HGROUP,4X8HSTANDARC)                     815.
0050      231 FORMAT(1H ,7H NUMBER,5H   I   ,4HNAME,4H I   ,10HDIFFICULTY,5X5HERRCR 816.
             1,3X10H   CHANGE  ,4H I   ,5HGRCUP,3X7HABILITY,5X5HERRCR/1X84('-'))         817.
0051      132 FCRMAT(1H ,18X25HITEM CHARACTERISTIC CURVE11X27HDEPARTURE FROM EXP 818.
             1ECTED ICC,18X16HFIT MEAN SQUARE /1X129('-'))                                 819.
0052      232 FCRMAT(1H ,12H SEC   ITEM  I,2(38H   1ST    2ND    3RD    4TH    5TH    6T 820.
             1H  I) ,'  WITHN   BETWN'            DISC  PCINT   I')                         821.
0053      332 FORMAT(1H ,12H NUM   NAME  I,6(6H GRCUP),2H I,6(6H GRCUP),2H I,            822.
             1'   GRCUP    GRCUP   TCTAL     INCX   DISER  I'/1X129('-'))                    823.
0054      133 FCRMAT(/12X'SERIAL CRCER',23X,'DIFFICULTY CRCER',21X,'FIT CRDER',        824.
             1/1X125('-')/                                                                  825.
             13(2X,'SEC ITEM',4X,'ITEM',4X,'DISC',4X,' FIT',' I'),2X,'PCINT'/          826.
             2      3(2X,'NUM NAME',4X,'DIFF',4X,'INCX',4X,'MN SC I'),2X,'DISER'/       827.
             31X,125('-'))                                                                  828.
0055          END                                                                           829.


0001          SUBROUTINENEWT(D,I,AB,IB,IS,NSEL,MINSC,MAXSC,IVAL,CRIT,LIM,SUM,SE) 736.
0002            DIMENSICN AB(1),IB(1),NSEL(1)                                            737.
        C**NEWTCN-RAPHSCN ITERATICN RCUTINE                                              738.
0003          TEMP=100.0                                                                   739.
0004          DC 2 IK=1,LIM                                                                740.
0005          SE=0.0                                                                       741.
0006          SUM=0.C                                                                      742.
0007          DC 1 K=MINSC,MAXSC                                                           743.
0008          IF(IVAL) 5,5,4                                                               744.
0009        4 IF(NSEL(K)-IVAL) 1,5,1                                                       745.
0010        5 P=EXP(AB(K)-D)                                                               746.
0011          P=P/(1.+P)                                                                   747.
0012          SE=SE+IB(K)*P*(1.-P)*I                                                       748.
0013          SUM=SUM+IB(K)*P*I                                                            749.
0014        1 CONTINUE                                                                     750.
0015          SSM=(IS-SUM)/SE                                                              751.
0016          IF(ABS(SSM).GT.TEMP) GC TC 6                                                 752.
0017   7      TEMP =ABS( SSM)                                                              753.
0018          D=D-SSM                                                                      754.
0019          IF(TEMP-CRIT)3,2,2                                                           755.
0020        2 CCNTINUE                                                                     756.
0021        3 RETURN                                                                       757.
0022     6    WRITE(6,100)IK,IS,SUM,SE,SSM,TEMP,D                                          758.
0023   100    FCRMAT(' CYCLE',I3,'  SCCRE',I4,' EXP SCCRE',F6.2,'   SE',F6.2,          759.
             1'  CCRRECTION',F6.3,'  PREV CORR',F6.3,'  DIFF',F6.3)                        760.
0024          SSM =SSM/2.0                                                                 761.
0025          GC TC 7                                                                      762.
0026          END                                                                          763.
```

```
FORTRAN IV G1   RELEASE 2.0              PROX                 DATE = 76302          00/16/35

                 SUBROUTINE PROX(IS,IB,DIFF,ABIL,ISEL,SE,N)
                 DIMENSION IS(1),IB(1),DIFF(1),ABIL(1),ISEL(1),SE(1)
                 COMMON NITEM,NGROF,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11),
                1SNAME(1)
                 L1 = LREC - 1
                 DDOT = 0.0
                 DO 2 I=1,NITEM
                 DIFF(I)=0.0
                 IF(ISEL(I))2,2,1
         1       DIFF(I)=NSUBJ*ISEL(I) - IS(I)
                 DIFF(I) = ALOG(DIFF(I)/IS(I))
                 DDOT = DDOT + DIFF(I)*ISEL(I)
         2       CONTINUE
                 D = 0.0
                 IX = NITEM + LREC
                 DDOT = DDOT/LREC
                 DO 4 I=1,NITEM
                 IX = IX + 1
                 IF(ISEL(I))4,4,3
         3       DIFF(I) = DIFF(I) - DDOT
                 D = D + DIFF(I)*DIFF(I)*ISEL(I)
                 SE(IX) = DIFF(I)
         4       CONTINUE
                 BDOT = 0.0
                 D = D/(2.89*(LREC-1))
                 B = 0.0
                 DO 5 J=1,L1
                 ABIL(J) = J
                 ABIL(J) = ALOG(ABIL(J)/(LREC-J))
                 BDOT = BDOT + ABIL(J)*IB(J)
         5       B = B + ABIL(J)*ABIL(J)*IB(J)
                 BDOT = BDOT/NSUBJ
                 B=B-BDOT*BDOT*NSUBJ
                 B=B/(2.89*(NSUBJ-1))
                 C = B*D
                 DDOT = 1.0 - C
                 X = (1.0+D)/DDOT
                 Y = (1.0+B)/DDOT
         7       B = SQRT(Y)
                 D = SQRT(X)
                 WRITE(6,101) B,D
       101       FORMAT('0   DIFFICULTY SCALE FACTOR',F6.2,
                1'   ABILITY SCALE FACTOR',F6.2)
                 DO 9 I=1,NITEM
                 IF(ISEL(I))9,9,8
         8       DIFF(I) = B*DIFF(I)
                 DIFF(I+NITEM) = DIFF(I)
                 Z=B*NSUBJ*ISEL(I)/(IS(I)*(NSUBJ*ISEL(I)-IS(I)))
                 SE(I) = SQRT(Z)
         9       CONTINUE
                 DO 10 J=1,L1
                 ABIL(J)=D*ABIL(J)
                 Z = D*LREC/(J*(LREC-J))
        10       SE(J+NITEM)=SQRT(Z)
                 RETURN
                 END
```

```
FORTRAN IV G1  RELEASE 2.0            REDOP                 DATE = 76302          00/16/39

0001            SUBROUTINE REDOP(IDATA,IB,IS,ISEL,MATX,IA,ID,DIFF)                      886.
0002            DIMENSION IDATA(1),IB(1),IS(1),ISEL(1),MATX(1),IA(1),ID(1),ICPT(8)      887.
0003            DIMENSION ISM(3),DIFF(1)                                                888.
0004            COMMON  NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)              889.
               1,SNAME(1)                                                               890.
0005            DATA INC/1H4/,ISM/'S','I','M'/                                          891.
0006            ISW1=ISW(1)                                                             892.
0007            IF(ISW1.EQ.0)ISW1=3                                                     893.
0008            NSUBJ=1                                                                 894.
0009             LLIM=ISW(2)                                                            895.
0010            KLIM=ISW(3)                                                             896.
      C**READ COLUMN SELECT CARD                                                        897.
0011            READ(5,101) (IA(I),I=1,LREC)                                            898.
0012        100 FORMAT(80I1)                                                            899.
      C**READ KEY CARD                                                                  900.
0013            READ(5,101)(ID(I),I=1,LREC)                                             901.
      C**READ OPTION LABEL CARD                                                         902.
0014            READ(5,101) ICPT                                                        903.
0015        101 FORMAT(80A1)                                                            904.
      C**PRINT SUMMARY INFORMATION                                                      905.
0016            WRITE(6,200)                                                            906.
0017        200 FORMAT(1H+,//1X16HCOLUMNS SELECTED/)                                    907.
0018            WRITE(6,201)(MN,MN=1,8)                                                 908.
0019        201 FORMAT(1H+,8I10)                                                        909.
0020            WRITE(6,202)(IND,N=1,8),(IND,N=1,63)                                    910.
0021        202 FORMAT(1H ,1H1,8A1,1H0,7(9A1,1H0))                                      911.
0022            WRITE(6,204)(IA(I),I=1,LREC)                                            912.
0023        204 FORMAT(1H ,80A1/)                                                       913.
0024            WRITE(6,206)(ID(I),I=1,LREC)                                            914.
0025        206 FORMAT(1H0,3HKEY/1X80A1)                                                915.
0026            N=1                                                                     916.
0027            NSC=1                                                                   917.
0028            ISW4 = ISW(8)                                                           918.
0029            CALL TRANS(IA,LREC)                                                     919.
0030            IF(ISW4.NE.0) CALL TRANS(ID,LREC)                                       920.
      C**COUNT ITEMS SELECTED                                                           921.
0031            DO 22 I=1,LREC                                                          922.
0032            KK=IA(I)                                                                923.
0033            IF(KK      )20,22,21                                                    924.
0034      20    NSC=NSC+1                                                               925.
0035         21 ISEL(N)=ID(I)                                                           926.
0036            IS(N)=C                                                                 927.
0037            N=N+1                                                                   928.
0038         22 CONTINUE                                                                929.
0039            N=N-NSC                                                                 930.
0040            IC=0                                                                    931.
0041            K=1                                                                     932.
0042            DO 1 I=1,LREC                                                           933.
```

```
0043            JX=IA  (I)
0044            IF(JX)19,19,17
0045      17    IF(ISW4.NE.1)JX=1
0046            DO 18 J=1,JX
0047            IC=IC+1
0048      18    IB(IC)=0
0049       19   DO 1 J=1,6
0050            MATX(K)=0
0051          1 K=K+1
0052            NLOW=0
0053            NHIGH=0
      C**READ AND WRITE FIRST SUBJECT
0054            READ(IS#1,101)(ID(I),I=1,LREC)
0055            WRITE(6,205)(ID(I),I=1,LREC)
0056        205 FORMAT(1H0,13HFIRST SUBJECT/1X80A1/)
0057            ASSIGN 95 TO ISIM
0058            DO 51 I=1,3
0059            IF(IC    (I).NE.ISM(I)) GO TO 2
0060      51    CONTINUE
0061            ASSIGN 94 TO ISIM
0062            CALL RESIN(IC,IDATA,DIFF,ISEL,1)
0063          2 DO 13 I=1,NITEM
0064         13 IDATA(I)=0
      C**SCORE EACH ITEM
0065            K=1
0066      4     DO66 I=1,LREC
0067            IF(IA(I))665,66,6
0068          6 DO 64 J=1,5
      C**COUNT OPTION SELECTED
0069            IF(IC(I)-ICPT(J))64,63,64
0070         63 L=6*(K-1)+J
0071            GO TO 65
0072         64 CONTINUE
0073      665   L=6*K
0074         65 MATX(L)=MATX(L)+1
0075            K=K+1
0076         66 CONTINUE
0077            CALL SCORE(IDATA,IC,IA,ISEL,NSC,ISW4)
      C**DISCARD ZERO OFPERFIT SCORES
0078      5     IF(NSC) 95,10,7
0079          7 IB(NSC)=IB(NSC)+1
0080            IF(NSC-IC)8,11,11
      C**WRITE SCRATCH FILE
0081          8 WRITE(1    )(IDATA(I),I=1,NITEM),(IC(I),I=LLIM,KLIM),NSC
0082            NSUBJ=NSUBJ+1
0083            IF(NSC-MINSC)95,87,87
0084         87 IF(NSC-MAXSC)88,88,95
      C**ACCUMULATE MARGINALS FOR CALIBRATION ROUTINES
```

```
FORTRAN IV G1  RELEASE 2.0              REDOP                DATE = 76302          00/16/35

0085          88 DO 9 I=1,NITEM
0086           9 IS(I)=IS(I)+IDATA(I)
0087             GO TO ISIM,(94,95)
0088         94  CALL RESIM(IC,ICATA,DIFF,ISEL,2)
0089             GO TO 96
0090          95 CONTINUE
           C**READ NEXT RECORD
0091             READ(ISW1,101,END=12)(ID(I),I=1,LREC)
           C**TEST FOR END OF FILE
0092       96    IF(  ID(1)-IND)2,12,2
0093          10 NLOW=NLOW+1
0094             GO TO ISIM,(94,95)
0095          11 NHIGH=NHIGH+1
0096             GO TO ISIM,(94,95)
           C**WRITE SUMMARY INFORMATION
0097          12 NSUBJ=NSUBJ-1
0098             WRITE(6,107)NITEM,NSUBJ
0099         107 FORMAT(//'ONUMBER OF ITEMS',I5/' NUMBER OF SUBJT',I5)
0100             CALL PAGE(2,I,J)
0101         103 FORMAT(//10X,'ALTERNATIVE RESPONSE FREQUENCIES'//
                1 2X,'SEQ',1X,'ITEM',6XA4,4(1XA4),2X,'UNKN',3X,'KEY'/
                2 2X,'NUM',1X,'NAME'/1X,53(1H-))
0102         104 FORMAT(I5,1X,A4,5X,'I',6I5,' I',2XA4,I6)
0103             L=1
0104             KK=6
           C**PRINT OPTION FREQUENCY TABLE
0105             WRITE(6,103) IOPT
0106             I=1
0107             DO 114 M=1,LREC
0108             IF(IA(M))113,114,112
0109       112   WRITE(6,104)I,SNAME(I),(MATX(K),K=L,KK),ISEL(I)
0110         113 L=KK+1
0111             KK=KK+6
0112             I=I+1
0113         114 CONTINUE
0114             WRITE(6,105)
0115       14    NSC=NLOW+NHIGH+NSUBJ
0116             CALL PAGE(2,I,J)
0117             WRITE(6,102) NLOW,NHIGH
0118         102 FORMAT(// 1X21HNUMBER OF ZERO SCORES I11/1X24HNUMBER OF PERFECT SC
                1ORES,I8)
0119             L=0
0120             DO725I=1,LREC
0121             KK=IA(I)
0122             IF(KK)      752,725,752
0123         752 L=L+1
0124             IF(ISW4.NE.1)KK=1
0125             ISEL(L)=KK
```

FCRTRAN IV G1 RELEASE 2.0 RELOF DATE = 76302 CC/16/35

```
  725 CONTINLE
      LREC=IC
      NITEM=L
      IC=N
      WRITE(6,106)N,L
  106 FCRMAT(//' NUMBER OF ITEMS SELECTEC',I4/' NUMBER OF ITEMS NAMEC',I
     17///1X,32('-'))
  105 FCRMAT(1X,53(1H-))
      END FILE 1
      REWINC 1
      RETURN
      END
```

```
FORTRAN IV G1  RELEASE 2.0            RESIM              DATE = 76302         00/16/35

0001                SUBROUTINE RESIM(ID,IDATA,DIFF,ISEL,M)
0002                DIMENSION ID(1),IDATA(1),DIFF(1),ISEL(1),ICODE(36),CCM(1)
0003                COMMON  NITEM,NGROUP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)
                   1,SNAME(1)
0004                DATA JD/' '/,IND/'*'/,ICODE/'0','1','2','3','4','5','6','7','8',
                   1'9','A','B','C','D','E','F','G','H','I','J','K','L','M','N','O',
                   2'P','Q','R','S','T','U','V','W','X','Y','Z'/
0005                GO TO (1,4),M
0006          1     READ(5,101) WIDTH,     ISUBJ,GMEAN,SD,ISED
0007        101     FORMAT(F5.0, I5,2F5.0, I10)
0008                IF(ISED.GT.0)ISEED=ISED
0009                WRITE(6,105)ISUBJ,WIDTH,GMEAN,SD,ISEED
0010        105     FORMAT('0SIMULATION OF',I5,' SUBJECTS',/,' TARGET VALUES --'
                   1,'TEST WIDTH =',F6.3,/,' ABILITY MEAN =',F6.3,' STANDARD',
                   2' DEVIATION =',F6.3,/,' SEED FOR RN GENERATOR =  ',I10,/)
0011                WIDTH= WIDTH/(NITEM-1)
0012                AB=0.
0013                P=0.
0014                DO 10 I=1,NITEM
0015                AB=AB+ISEL(I)
0016         10     P=P-I*ISEL(I)
0017                DIFF(1)=WIDTH*(1.+P/AB)
0018                I=1
0019                WRITE(6,102)I,DIFF(I)
0020        102     FORMAT(' ITEM NUMBER',I5,' DIFFICULTY=',F6.3)
0021                DO 2 I=2,NITEM
0022                DIFF(I)=DIFF(I-1)+WIDTH
0023                WRITE(6,102)I,DIFF(I)
0024          2     DIFF(I-1)=EXP(DIFF(I-1))
0025                DIFF(NITEM)=EXP(DIFF(NITEM))
0026                DO 3 I=1,150
0027          3     ID(I)=JD
0028                ICNT=0
0029                ABM=0.
0030                ABM2=0
0031          4     IF(ICNT.LT.ISUBJ) GO TO 5
0032                ID(1)=IND
0033                ABM2=SQRT((ABM2-ABM*ABM/ICNT)/(ICNT-1))
0034                ABM=ABM/ICNT
0035                WRITE(6,104)ICNT,ABM,ABM2
0036        104     FORMAT(1H0,I5,' SUBJECTS SIMULATED.   MEAN ABILITY ='
                   1,F6.3,' STANDARD DEVIATION =',F6.3)
0037                RETURN
0038          5     ICNT=ICNT+1
0039                CALL GGNN(ISEED ,1,CCM)
              C  GGNN GENERATES NORMALL VARIATES WITH MEAN 0 AND VARIANCE 1.
              C      ISEED IS THE SEED FOR THE GENERATOR WHICH IS READ FROM
              C             SIMULATION DESCRIPTION CARD.
```

```
                C       '1' INDICATES ONLY ONE DEVIATE IS TO BE RETURNED.                  1050.
                C       COM IS AN ARRAY IN WHICH THE RANDOM NUMBER IS RETURNED.            1051.
0040                  ABL=SD*COM(1)+GMEAN                                                  1052.
0041                  AEM=AEM+ABL                                                          1053.
0042                  ABM2=ABM2+ABL*ABL                                                    1054.
0043                  AB=EXP(ABL)                                                          1055.
0044                  DO 8 I=1,NITEM                                                       1056.
0045                  IX=ISEL(I)                                                           1057.
0046                  IF(IX.EQ.0) GO TO 8                                                  1058.
0047                  P=EXP(AB-DIFF(I))                                                    1059.
                C  NEITHER GGUN NOR GGU3 ARE INCLUDED IN THE RASCH SOURCE DECK.           1059.3
                C  THEY ARE PART OF 'SYS2.IMSL' PACKAGE AT THE UNIVERSITY OF CHICAGO.    1059.4
0048                  P=P/(1.+P)                                                           1100.
0049                  NSC=0                                                                1101.
0050                  DO 7 J=1,IX                                                          1102.
0051                  CALL GGU3(ISEED,1,COM)                                               1103.
                C  GGU3 GENERATES UNIFORM RANDOM NUMBERS. THE ARGUMENT LIST IS             1104.
                C       IS IDENTICAL TO 'GGUN.'                                            1105.
0052                  IF(COM(1)-P)6,6,7                                                    1106.
0053            6     NSC = NSC +1                                                         1107.
0054            7     CONTINUE                                                             1108.
0055                  ID   (I)=ICODE(NSC+1)                                                1109.
0056            8     CONTINUE                                                             1110.
0057                  IF(ISW(11).NE.0)WRITE(6,103)ICNT,ABL,(ID(I),I=1,NITEM)               1111.
0058            103   FORMAT(' SUBJECT NUMBER',I5,' ABILITY ',F6.3,' RESPONSES ',80A1)     1112.
0059                  RETURN                                                               1113.
0060                  END                                                                  1114.
```

```
0001            SUBROUTINE SCORE(IDATA,IC,ISEL,KEY,NSC,M)
0002            DIMENSION IDATA(1),IC(1),ISEL(1),KEY(1)
0003            COMMON  NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,ID,KCAB,ISW(11)
               1,SNAME(1)
0004            DATA BLK/' '/,AST/'*'/
0005            MM=M+1
0006            NSC=0
0007            J=0
0008            GO TO (9,1,14,19,24),MM
0009          1 CALL  TRANS(ID,LREC)
0010            DO 5 I=1,LREC
0011            K=ISEL(I)
0012            IF(K)4,5,2
0013        2   IF(K-ID(I))6,3,3
0014        3   NSC=NSC+ID(I)
0015        4   J=J+1
0016            IDATA(J)=ID(I)
0017        5   CONTINUE
0018            RETURN
0019        6   WRITE(6,101)(ID(K),K=1,LREC)
0020            WRITE(6,102)(BLK,K=1,I      ),AST
0021      101   FORMAT(1X,'ILLEGAL CODE.  CASE OMITTED'/(1X80I1))
0022      102   FORMAT(81A1)
0023            NSC=-1
0024            RETURN
0025        9   J=1
0026            DO 13 I=1,LREC
0027            K=ISEL(I)
0028            IF(K)12,13,10
0029       10   IF(IC(I)-KEY(J))12,11,12
0030       11   NSC=NSC+1
0031            IDATA(J)=1
0032       12   J=J+1
0033       13   CONTINUE
0034            RETURN
0035       14   CALL  TRANS(IC,LREC)
0036            J=1
0037            DO 18 I=1,LREC
0038            K=ISEL(I)
0039            IF(K)17,18,15
0040       15   IF(IC(I)-KEY(J))16,16,17
0041       16   NSC=NSC+1
0042            IDATA(J)=1
0043            J=J+1
0044            GO TO 18
0045       17    IDATA(J)=0
0046            J=J+1
0047       18   CONTINUE
```

```
              RETURN
      19      CALL  TRANS(IC,LREC)
              J=1
              DO 23 I=1,LREC
              K=ISEL(I)
              IF(K)22,23,20
      20      IF(IC(I)-KEY(J))22,21,21
      21      NSC=NSC+1
              IDATA(J)=1
               J=J+1
              GO TO 23
      22      IDATA(J)=0
              J=J+1
      23      CONTINUE
              RETURN
      24      CONTINUE
      C  THIS SPACE RESERVED FOR USER SUPPLIED LOGIC.
              RETURN
              END
```

```
FORTRAN IV G1  RELEASE 2.0            SMMRY              DATE = 76302          00/16/35

0001              SUBROUTINE SMMRY(DIFF,A,IDATA,IS,MATX,ISEL,B,IB)                      1182.
0002              DIMENSION DIFF(1),ISEL(1),IDATA(1),MATX(1),IS(1),A(1), B(1),IB(1)    1183.
0003              DIMENSION XX(3),STAT(6),AVG(3),AABB(3)                                 1184.
0004              COMMON  NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)             1185.
                 1,SNAME(1)                                                              1186.
0005              LLINE=0                                                                1187.
            C**CLEAR NECESSARY ARRAYS                                                    1188.
0006              DO 97 I=1,NITEM                                                        1189.
0007              IF(ISEL(I))97,97,95                                                    1190.
0008           95 K=I                                                                    1191.
0009           97 CONTINUE                                                               1192.
0010              DO 98 I=1,3                                                            1193.
0011      98      AVG(I)=0.0                                                             1194.
0012              DO 99 I=1,6                                                            1195.
0013           99 STAT(I)=0.0                                                            1196.
0014              CALL PAGE(2,I,J)                                                       1197.
0015              DO 1 I=1,NITEM                                                         1198.
0016              IF(ISEL(I))10,10,1                                                     1199.
0017      10      DIFF(I)=0.0                                                            1200.
0018              A(I)=0.0                                                               1201.
0019              MATX(I)=0                                                              1202.
0020            1 IS  (I)=100.0*DIFF(I)                                                  1203.
            C**FIND DIFFICULTY AND FIT ORDERS                                            1204.
0021              CALL SRT(IS  ,IDATA)                                                   1205.
0022              CALL SRT(IB,IS)                                                        1206.
            C**PRINT HEADINGS                                                            1207.
0023              WRITE(6,100)                                                           1208.
0024          100 FORMAT(/12X'SERIAL ORDER',23X,'DIFFICULTY ORDER',21X,'FIT ORDER',     1209.
                1/1X119('-')/2(2X,'SEQ ITEM',4X,'ITEM',4X,'DISC',4X,' FIT  I'),         1210.
                12X,'SEQ ITEM',4X,'ITEM',4X,'DISC',4X,' FIT ',3X,'POINT  I'/            1211.
                2     2(2X,'NUM NAME',4X,'DIFF',4X,'INDX',4X,'MN SQ I'),                1212.
                32X,'NUM NAME',4X'DIFF',4X'INDX',4X'MN SQ',3X'BISER  I'/1X119('-'))     1213.
            C**PRINT TABLE WITH I EQ SEQ ORDER, J EQ DIFF ORDER AND K EQ FIT ORDER       1214.
0025              DO22 I=1,NITEM                                                         1215.
0026              J=IDATA(I)                                                             1216.
0027              K=IS(I)                                                                1217.
0028              AABB(1)= IB(I)/100.0                                                   1218.
0029              AABB(2)= IB(J)/100.0                                                   1219.
0030              AABB(3)= IB(K)/100.0                                                   1220.
0031              CALL PAGE(4,3,LLINE)                                                   1221.
0032              WRITE(6,101) I,SNAME(I),DIFF(I),A(I),AABB(1),J,SNAME(J),DIFF(J),       1222.
                 1A(J),AABB(2),K,SNAME(K),DIFF(K),A(K),AABB(3),B(K)                      1223.
0033          101 FORMAT(2(I5,1XA4,3F8.2,2X1H I),I5,1XA4,4F8.2,'    I')                  1224.
0034              XX(1)=DIFF(I)                                                          1225.
0035              XX(2)=A(I)                                                             1226.
0036              XX(3)=AABB(I)                                                          1227.
0037              KL=1                                                                   1228.
0038              IF(ISEL(I))22,22,5                                                     1229.
```

```
      5 DO 2 K=1,3
        AVG(K)=AVG(K)+XX(K)*ISEL(I)
        DO 2 L=1,K
        STAT(KL)=STAT(KL)+XX(K)*XX(L)*ISEL(I)
      2 KL=KL+1
        IF(ISW(7)) 22,22,21
21      WRITE(7,103)J,SNAME(I),XX
103     FORMAT(I3,A4,3F7.3)
     22 CONTINUE
        KL=1
        X=   IC -1
C**COMPLTE MEANS, STANDARD DEVIATIONS AND CORRELATIONS
        DO 3 I=1,3
        AVG(I)=AVG(I)/ LREC
        DO 3 J=1,I
        STAT(KL)=(STAT(KL)-LREC *AVG(I)*AVG(J))/(LREC-1)
      3 KL=KL+1
        KL=1
        DO 4 I=1,3
        K=(I*(I+1))/2
        XX(I)=SQRT(STAT(K))
        DO 4 J=1,I
        STAT(KL)=STAT(KL)/(XX(I)*XX(J)    )
      4 KL=KL+1
        WRITE(6,102) AVG,XX,STAT(2),STAT(4),STAT(5)
    102 FORMAT(1X,119(1H-)/ 6X'MEAN',3F8.2/ 6X,'S.D.',3F8.2,6X, 'CORRELATI
       1ON',6X'DIFF*DISC=',F6.2,6X,'DIFF*MASC=',F6.2,6X,'DISC*MASC=',F6.2)
        RETURN
        END
```

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```
FORTRAN IV G1  RELEASE 2.0           SRT                 DATE = 76302          00/16/35

0001             SUBROUTINE SRT(MATX,IDATA)                                            1259.
0002             DIMENSION MATX(1),IDATA(1)                                            1260.
0003             COMMON  NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)            1261.
                1,SNAME(1)                                                             1262.
           C**SORT CONTENTS OF MATX, STORE ORDER IN IDATA                              1263.
0004             DO 1 I=1,NITEM                                                        1264.
0005           1 IDATA(I)=I                                                            1265.
0006             DO 2 I=1,NITEM                                                        1266.
0007             DO 30 J=I,NITEM                                                       1267.
0008             IX=IDATA(I)                                                           1268.
0009             JX=IDATA(J)                                                           1269.
0010             IF(MATX(JX)-MATX(IX))3,30,30                                          1270.
0011           3 IDATA(I)=JX                                                           1271.
0012             IDATA(J)=IX                                                           1272.
0013          30 CONTINUE                                                              1273.
0014           2 CONTINUE                                                              1274.
0015             RETURN                                                                1275.
0016             END                                                                   1276.


0001             SUBROUTINE TRANS(IA,LENG)                                             1277.
0002             DIMENSION IA(1),ICODE(36)                                             1278.
0003             COMMON  NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)            1279.
                1,SNAME(1)                                                             1280.
0004             DATA ICODE/'0','1','2','3','4','5','6','7','8','9','A','B','C','D'   1281.
                1,'E','F','G','H','I','J','K','L','M','N','O','P','Q','R','S','T',     1282.
                2'U','V','W','X','Y','Z'/,BLK/' '/,IAMP/'&'/                           1283.
0005             DO 18 I=1,LENG                                                        1284.
0006             K=IA(I)                                                               1285.
0007             IF(K.NE.BLK) GO TO 14                                                 1286.
0008             IA(I)=0                                                               1287.
0009             GO TO 18                                                              1288.
0010      14     IF(K.NE.IAMP) GO TO 15                                                1289.
0011             IA(I)=-1                                                              1290.
0012             GO TO 18                                                              1291.
0013      15     DO 17 J=1,36                                                          1292.
0014             IF(K-ICODE(J))17,16,17                                                1293.
0015      16     IA(I)=J-1                                                             1294.
0016             GO TO 18                                                              1295.
0017      17     CONTINUE                                                              1296.
0018             IA(I)=0                                                               1297.
0019      18     CONTINUE                                                              1298.
0020             RETURN                                                                1299.
0021             END                                                                   1300.
```

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```
LEASE 2.0              UCON                 DATE = 76302          00/16/35
      SUBROUTINE UCON(S,N,D,AB,ISEL,SE)
      INTEGER R(1),S(1),ISEL(1)
      DIMENSION D(1),AB(2),SE(1)
      COMMON   NITEM,NGROP,MINSC,MAXSC,LREC,NSUBJ,IC,KCAB,ISW(11)
     1,SNAME(1)
      L=NITEM
      L1=LREC-1
      DX=LREC
      DX=DX/L1
      DO 199 I=1,L
      IF(ISEL(I).EQ.0) GO TO 199
      D(I) = D(I)*DX
  199 CONTINUE
      DO 2000 IT=1,10
      NITER = IT
      CRIT=0.0
      CEN=0.0
      DO 201 I=1,L
      IF(ISEL(I)) 201,201,200
  200 CONTINUE
      CALLNEWT(D(I),ISEL(I),AB,F, S(I),NSEL,MINSC,MAXSC,0.,.05,10,DD,P)
      CEN=CEN+D(I)*ISEL(I)
      SE(I)=F
  201 CONTINUE
      CEN=CEN/LREC
      DO 212 I=1,L
      D(I)=D(I)-CEN
      IX=I+L+LREC
      IF(ISEL(I)) 212,212,211
  211 CRIT=CRIT+ABS(D(I)-SE(IX ))
  212 CONTINUE
       IF(IT.GT.1) GO TO 205
      DO 204 I=1,L
      IF(ISEL(I))204,204,202
 202  D(I+2*NITEM)=D(I)/DX
 204  CONTINUE
 205  IF((CRIT/IC)-0.025)330,1999,1999
 1999 DO 214 K=MINSC,MAXSC
      DO 215 ITK=1,5
      IX=K+L
      SE(IX )=0.0
      DD=0.0
      DO 216 I=1,L
      IF(ISEL(I))216,216,213
  213 P=EXP(AB(K)-D(I))
      P=P/(1.0+P)
      SE(IX )=SE(IX )+P*(1.0-P)*ISEL(I)
      DD=DD+P*ISEL(I)
```

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```
FORTRAN IV G1  RELEASE 2.0          UCJN          DATE = 76302          00/16/35

  216 CONTINUE
      CC=(K-CC)/SE(IX)
      AB(K)=AB(K)+CC
      IF(ABS(CC)-0.05)214,215,215
  215 CONTINUE
  214 CONTINUE
      IX=L+LREC
      DO 2000I=1,L
      IX=IX+1
      SE(IX)=D(I)
 2000 CONTINUE
  330 IX=L+LREC
      CC=LREC-1
      DD=DC/LREC
       IT=LREC-1
      DO 332 I=1,L
      IX = IX + 1
      SE(IX)=D(I)-SE(IX)
 332  C(I)=D(I)/DX
      WRITE(6,100)NITER
 100  FORMAT('  NUMBER OF ITERATIONS = ',I3)
      RETURN
      END
```

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APPENDIX C

BICAL CONTROL CARDS

Position	Name		Format and Description
1	Title card		(20A4) Descriptive heading to be printed at the top of each page of output
2	Data Definition		(14I5)
	CC	Label	Definition
	1 - 5	NITEM	Total number of items to be read before deletions. This is equal to the number of non-zero entries on the column select card and is the number of item names expected.
	6 - 10	NGROP	Smallest allowable average group size for testing item fit. This is used to determine the number of score groups. The same value is used to terminate execution before estimation if the total number of subjects is less than NGROP.
	11 - 15	MINSC	Minimum score to be included in the calibration sample.
	16 - 20	MAXSC	Maximum score to be included.
	21 - 25	LREC	Number of columns in the input record to be scanned. It must be large enough to cover all columns containing items and also to skip any extra cards in the subject record.

26 - 30	KCAB	Calibration code 1 = Normal approximation method, should be used with long tests and symmetrical distribution of scores. 2 = Corrected unconditional maximum likelihood estimation. Should be used with shorter tests and skewed distributions.
31 - 35	KSCOR	Scoring code b,0 = score dichotomously according to KEY 1 = data already scored 2 = score dichotomously, correct if X $\leq$ KEY 3 = score dichotomously, correct if X $\geq$ KEY
36 - 40	INFLE	Alternative input file unit number b,0 = Unit 5.
41 - 45	LLIM	Alternative output file--start of identification field in record
46 - 50	KLIM	Alternative output file--end of identification field in record. If LLIM and KLIM are 1 and LREC the entire record will be copied as the identification.
51 - 55	NUFLE	Alternative output file logical unit number. For each valid input record, a new record will be generated containing raw score, scaled ability in logits and the identification field defined by LLIM and KLIM.
56 - 60	KPRTR	Control switch for optional output b,0 Print all plots 1 Omit score histogram 2 Omit Fit plots 3 Omit both
61 - 65	KSIM	Print simulated persons if > 0

66 - 70	KDIFF	Punch item statistics on unit 7. Output insists of item sequence number, item name, difficulty, discrimination and total fit mean square. Format is (I3,A4,3F7.3).
3.	Item Name Card(s)	(20A4) A four character alphanumeric name for each of the "NITEM" items.
4.	Column Selection	(80A1) A record identical in size to each person's record indicating how the data in that position is to be used. For each position b,0 = skip column 1-9 = include item in corresponding column. Maximum allowable code is 1-9 as given. A-Z= include item in corresponding column. Maximum allowable code is 10-35 (A=10, etc.) &= delete item in corresponding column after reading names.
5.	Scoring Key	(80A1) Corresponds to perfect input record. It must be included regardless of KSCOR.
6.	Options Label	(5A1) Identifies up to five option labels for which the number of occurrances will be counted for each item.
(7)	Datq cards	
(7 a)	End of Data	* in col. 1.
(8)	Simulation header	SIMULATE in columns 1-8 causes program to simulate data rather then read. If included it must be followed by

(9) Simulation task description card (F5.0,15,2F5.0,110)

C.C.	Label	Definition
1 - 5	WIDTH	Range of item difficulties to be generated.
6 - 10	ISUBJ	Number of persons to be generated.
11 - 15	GMEAN	Mean ability of population sampled.
16 - 20	SD	Standard deviation of ability.
21 - 25	ISED	Seed for random number generator-should only be coded for first generation in each run.

(10) End of job — **** in columns 1-4
Program will keep recycling looking for new problems until this card is encountered. As many jobs as desired may be stacked.

University of Chicago JCL

```
//jjjj JOB (valid UC job card)    ,RE=129K
// EXEC PGM=BICAL
//STEPLIB DD DSN=$2DD130.S05.DATA(BICAL) ,DISP=SHR
//FT01F001 DD   UNIT=SYSCR,DISP=NEW,SPACE=(TRK,(5,1))
//FTxxF001 DD   alternative input file description
//FTyyF001 DD   alternative output file description
//FT07F001 DD   SYSOUT=B,DCB=(RECFM=FB,BLKSIZE=80)
//FT06F001 DD   SYSOUT=A,DCB=(RECFM=FA,BLKSIZE=133)
//FT05F001 DD*
```

The FT05 card is followed by the first job card. Cards FTxx, FTyy and FT07 are not always required.

Include FTxx if input records are not on cards. The xx should be replaced by the value of INFLE coded on the data description card (cc 36-40).

Include FTyy if a new output is to be produced. The yy should be replaced by the value of NUFLE (cc 51-55).

Include FT07 if item data is to be punched, (cc 66-70).